THE OPTIMAL POPULATION DISTRIBUTION ACROSS CITIES AND THE PRIVATE-SOCIAL WEDGE

DAVID ALBOUY AND NATHAN SEEGERT

Abstract. The conventional wisdom is that market forces cause cities to be inefficiently large, and public policy should limit city sizes. This wisdom assumes, unrealistically, that city sites are homogeneous, that land is given freely to incoming migrants, and that federal taxes are neutral. In a general model with city heterogeneity and cross-city externalities, we show that cities may be inefficiently small. This is illustrated in a system of monocentric cities with agglomeration economies in production, where cross-city externalities arise from land ownership and federal taxes. A calibrated model accounting for heterogeneity suggests that in equilibrium, cities may be too numerous and underpopulated.

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1. Introduction

Cities define civilization and yet are often perceived as too large. Positive urban externalities from human capital spillovers – seen by Lucas (1988) as the key to economic growth – and from greater matching and sharing opportunities (Duranton and Puga, 2004), provide the agglomeration economies that bind firms and workers together in cities. These centripetal forces are countered by centrifugal forces that keep the entire population from agglomerating into one giant megacity. Such centrifugal forces include the urban disamenities of congestion, crime, pollution, and contagious disease, all thought to increase with population size. Many economists, including Tolley (1974); Arnott (1979); Upton (1981); Abdel-Rahman (1988); Fenge and Meier (2002), have argued that because migrants to cities do not pay for the negative externalities that they cause, free migration will cause cities to become inefficiently large from a social point of view. This view is presented as fact in O'Sullivan's (2003) *Urban Economics* textbook, and is easily accepted as it reinforces ancient (e.g. Biblical) negative stereotypes of cities. Ultimately, this view provides support for policies to limit urban growth, such as land-use restrictions, and disproportionate federal transfers towards rural areas.

The canonical argument explaining why cities are too large is analogous to the argument explaining why free-access highways become overly congested, first presented in Knight (1924). The cost migrants pay to enter a city is equal to the social average cost rather than the social marginal cost. This is illustrated in Figure 1, except that costs are translated to benefits using a minus sign. The social marginal benefit curve, drawn in terms of population, crosses the average benefit curve at its maximum, A, and thus the marginal benefit curve is lower than the average benefit curve beyond this size. Migrants, who ignore externalities and thus respond to the average benefit, will continue to enter a city until the average benefit of migration equals the outside option at B. This population level is only stable when benefits are falling with city size, and thus cities can never be too small.

The analogy of a city to a simple highway, which obviously appeals to urban economists, is misleading for three fundamental reasons. First, the land sites that cities occupy may differ in the natural advantages they offer to households and firms, such as a mild climate or proximity to water. Thus, in a multi-site economy it may be efficient to add population to an advantageous site beyond its isolated optimum at A when the alternative is to add population to an inferior site. Analogously, it makes sense to over-congest a highway when the alternative is a dirt road. Thus, the outside marginal benefit from residing in another city may be below the peak benefit at A, so that the social optimum is at a point such as C, where the social marginal benefit is equal to the lower outside benefit.

Second, access to a city and its employment or consumption advantages is not free: migrants must purchase land services and bear commuting costs to access these advantages. Thus, unlike a free-access highway, migrants must pay a toll to access a city's opportunities, and this toll is highest in cities offering the best opportunities. Thus many of the benefits of urbanization are appropriated by pre-existing land owners rather than by incoming migrants, whose incentive to move may be below the social average benefit.

Third, workers must pay federal taxes on their wage incomes, which increase with a city's advantages to firms but decrease with a city's advantages to households (Haurin, 1980; Roback, 1982). Thus, federal taxes create a toll that is highest in areas offering the most to firms and the least to households, slowing migration to these areas. These effects are modeled by Albouy (2009) with exogenous amenities, but are modeled here with amenities that are endogenous to city population. If urban size benefits firms but harms households, then federal taxes impose tolls that are highest in the largest cities, strongly discouraging migration to them.

Land income and federal taxes together drive a wedge between the private and social gains that accrue when a migrant enters a city. Migrants respond to the private average benefit, illustrated by the dotted line in figure 1, putting the city population at point E with free migration, or point D if migrants manage to maximize private benefits in the city. In this example, cities can be vastly undersized, producing a welfare loss seen as large as the shaded area.

To the extent that individuals pay for land services and federal taxes, payments to land and labor may be viewed as common resources. Because both rents and wages increase with city size, cities can be too small in a stable market equilibrium as migrants have no incentive to contribute to these common resources: migrants will artificially prefer less advantageous sites to avoid paying higher land rents and federal taxes. In essence, inter-city migration decisions involve cross-city fiscal externalities, which un-internalized, lead to inefficiently small cities. This may be amplified if big-city residents have greater positive net externalities than small-city residents for non-fiscal reasons, e.g. if big-city residents have lower greenhouse-gas emissions than small-city residents (Glaeser and Kahn, 2010).

We begin our argument in section 2 using a basic representation of cities, which may be viewed as clubs with external spillovers. In section 3 we provide a microeconomic foundation to this representation with a system of cities based on the monocentric-city model of Alonso (1964); Muth (1969); Mills (1967) to give form to our functions and concreteness to our simulations. Urban economies of scale are modeled through inter-firm productivity spillovers that lead to increasing returns at the city level, while urban diseconomies are modeled through generalized commuting costs.[1] In addition, city sites are heterogeneous in the natural advantages they provide to firms in productivity or to households in quality of life. This model is calibrated as realistically as possible to demonstrate the theoretical results concretely and to illustrate their plausibility in the reality. Section 4.3 improves on existing work by allowing the number of cities to vary, analyzing differences on the "extensive" margin, i.e. the number of sites occupied, as well as on the "intensive" margin, i.e. on how the population is distributed across a fixed number of occupied sites. The distribution of natural advantages across sites is modeled using Zipf's Law.

Throughout the analysis we consider four types of population allocations. We begin with the standard problem of how a city planner maximizes the average welfare of the inhabitants of a single city, ignoring the effects on the outside population and internalizing any cross-city externalities. Second, we consider the welfare optimum for an entire population, whereby a federal planner allocates individuals across heterogenous sites, determining the number and size of cities. We put particular emphasis on the case where individuals are equally well off in all cities, as would be implied by free mobility. Third, we look at the equilibrium that occurs when populations are freely mobile, but in a private ownership economy where they must rent land and pay federal taxes. Fourth, we consider political equilibria in a private ownership economy that could arise when local governments

[1] This model can be expanded to incorporate other realistic features of cities, e.g. non-central firm placement in Lucas and Rossi-Hansberg (2002), without losing the main point.

restrict population flows into their city, ignoring the effects on other cities. These four cases share a symmetry illustrated below:

	Multiple Authority	Single Authority
Planned Economy	*City Planner*	*Federal Planner*
Private Ownership	*Political Equilibrium*	*Competitive Equilibrium*

We find that the efficient population distribution tends to concentrate the population in the fewest number of cities, fewer than would be allocated by isolated city planners. Meanwhile, equilibrium forces disperse the population inefficiently, causing inferior sites to be inhabited, with local political control potentially exacerbating this problem. Examples throughout the paper are illustrated graphically using the calibrated model from section 3. The simulation, which allows the number of cities to be endogenous, demonstrates that there may be (roughly) 40 percent too many occupied sites, with welfare costs equal to 1 percent of GDP.

There is a substantive literature on systems of cities or regions, pioneered by Buchanan and Goetz (1972); Flatters, Henderson, and Mieszkowski (1974), developed extensively by Henderson (1977), and given comprehensive treatments by Fujita (1989); Abdel-Rahman and Anas (2004). Helpman and Pines (1980) argues that it is best to assume that households own a diversified portfolio of land across cities and model sites that differ in their inherent quality of life, but treat output per worker as fixed. Hochman and Pines (1997) model federal taxes in cities that offer different fixed wage levels.

Our work attempts to improve on this literature by carefully defining social and private benefits at both intra and inter-urban scale, and their associated solution concepts. The cities in the system are remarkably heterogeneous as they may differ in both natural advantages to firms (inherent productivity) and households (quality of life), and flexibly incorporate increasing returns to scale, through an arbitrary agglomeration parameter, and decreasing returns to scale, through an arbitrary commuting-cost parameter.[2] The quantitatively important institutions of land ownership

[2]The modeling of natural advantages helps to fill in a gap in the literature mentioned by Arnott (2004, p. 1072). regarding the Henry George Theorem:
"The HGT is derived on the assumption that land is homogenous, but in reality locations differ in terms of fertility, natural amenities such as visual beauty and climate, and natural accessibility such as access to the sea or a navigable river. How do these Ricardian differences in land affect the Theorem qualitatively? To my knowledge, this question has not been investigated in the literature."

and federal taxation are also simultaneously addressed. Perhaps the most interesting aspect of this research is that it provides some empirical content to an issue that has largely remained completely theoretical.

2. Basic Model

2.1. **Planned Economy.** A homogenous population, numbering N_{TOT}, must be allocated across a set of sites, $\bar{\mathcal{J}} = \{0, 1, 2, ..., \bar{J}\}$, indexed by j, with the population at each site given by N_j, such that

$$\sum_{j=0}^{\bar{J}} N_j = N_{TOT},\ N_j \geq 0 \text{ for all } j \tag{1}$$

The non-negativity conditions reflect that some sites may be uninhabited. The population allocation is written in vector form as $N = (N_0, N_1, ..., N_J)$. Assume that the social welfare function can be written as an additively separable function

$$W(N) = \sum_{j=0}^{\bar{J}} SB_j(N_j) \tag{2}$$

where $SB_j(N_j)$ is the social benefit, net of costs, of having N_j people living on site j, normalized such that an uninhabited site produces no benefit $SB_j(0) = 0$. The social benefit includes the value of goods produced by residents and the amenities they enjoy net of the disamenities they endure such as commuting costs. Some benefits only affect residents inside the city — such as climate amenities, transportation costs, or congestion – while others – such as global pollution, technological innovations, and federal tax payments – may affect residents of other cities. Region $j = 0$ is assumed to be a non-urban area with $SB_0(N) = b_0 N$, where b_0 is a constant.

By definition, the social average benefit of residing in city j, $SAB_j(N_j) \equiv SB_j(N_j)/N_j$. The social average benefit is assumed to be twice continuously differentiable, strictly quasi-concave, and

$$\frac{\partial SAB_j(N_j^{cp})}{\partial N} = 0 \text{ for some finite } N_j^{cp} > 0, \text{ for all } j \tag{3}$$

making the SAB_j function single-peaked. Urban scale economies dominate diseconomies for populations less than N^{cp} while the opposite holds for populations greater than N^{cp}. This single peak at N_j^{cp} designates the choice of a city planner (hence "cp") whose objective is to maximize the social average benefit within the city, assuming all city benefits are internalized. The social marginal benefit of residing in city j is given by the identity

$$SMB_j(N) \equiv \frac{\partial SB_j(N)}{\partial N} = SAB_j(N) + \underbrace{N\frac{\partial SAB_j(N)}{\partial N}}_{\text{Within-City Wedge}} \tag{4}$$

where the within-city wedge, the second term, captures the effect of an additional migrant on inframarginal inhabitants of city j through scale economies. Therefore SMB_j is larger than SAB_j when SAB_j is increasing, smaller than SAB_j when it is decreasing, and equal at N_j^{cp}.

$$SMB_j(N_j^{cp}) = SAB_j(N_j^{cp}) \tag{CP}$$

City planners are solely concerned with their city and do not coordinate with other city planners. An integer problem arises if the city planner optima do not add up to the total population, i.e. $\sum_{j\in\mathcal{J}} N_j^{cp} \neq N_{TOT}$, for $\mathcal{J} \subseteq \bar{\mathcal{J}}$. We focus here on situations where N_{TOT} is large relative N_j^{cp}, making integer problems unimportant.

The federal optimum, which determines the efficient population distribution, maximizes the social welfare in (2) subject to the constraints in (1). The necessary condition is given by

$$SMB_j(N_j^{fp}) = SMB_k(N_k^{fp}) = \mu \tag{FP}$$

across any two sites j and k that are inhabited, where $\mu \geq 0$ is the multiplier on the population constraint, and N_j^{fp} refers to the population chosen by the federal planner.

Conditions CP and FP characterize the city and federal planner equilibria on the intensive margin or how population is distributed across cities. This paper adopts the extensive margin, how many cities are created, algorithm created in Seegert (2011a) where planners inhabit and populate cities in a two-stage game. In the first stage planners decide (simultaneously for the city planners) which cities to create. In the second stage population is distributed according to conditions CP and FP respectively. The subgame perfect equilibrium of this dynamic game characterizes the extensive margin which can be found by backward induction.[3,4]

In the model cities are heterogenous in the amount of social benefit they produce for a given population level. Modeling systems of cities with heterogeneity is important because the city planner's system, which is often used in the literature, differs from the federal planner's when heterogeneity exists.[5]

DEFINITION: City j is *superior* to city k if $SB_j(N_i) > SB_k(N_i)$ for all N_i

RESULT 1: *When cities are heterogeneous, in that some cities are superior to others, the city planner optimum is not efficient.*

$$
\begin{aligned}
SMB_k(N_k^{cp}) &= SAB_k(N_k^{cp}) && \text{Definition } N_k^{cp}. \\
SAB_k(N_k^{cp}) &< SAB_j(N_k^{cp}) && \text{City } j \text{ superior to city } k. \\
&< SAB_j(N_j^{cp}) && \text{Definition } N_j^{cp}. \\
&= SMB_j(N_j^{cp}) && \text{Definition } N_j^{cp}. \\
&\Rightarrow SMB_k(N_k^{cp}) < SMB_j(N_j^{cp})
\end{aligned}
$$

COROLLARY 1: *When cities are heterogeneous, in that some cities are superior to others, the city planner optimum allocates too few people to superior cities.*

[3]The federal planner efficiently inhabits sites $\mathcal{J}^{fp}$ using a backward induction algorithm: for every $\mathcal{J} \subseteq \bar{\mathcal{J}}$, the efficient population allocation $\tilde{\mathbf{N}}^{fp}(\mathcal{J})$ can be determined using (FP), and the associated second-order conditions; then the $\mathcal{J}$ that maximizes $W[\tilde{\mathbf{N}}^{fp}(\mathcal{J})]$ over the power set, $P(\bar{\mathcal{J}})$, determines the solution $\mathcal{J}^{fp}$ and $\mathbf{N}^{fp} = \tilde{\mathbf{N}}^{fp}(\mathcal{J}^{fp})$.
[4]Given constraint (1) the federal planner chooses the efficient set of cities to inhabit and allocation across these cities, therefore the solution does not have an integer problem.
[5]However, when cities are homogeneous the planner systems coincide. To show this, let N^{cp} satisfy (3) for all, then by homogeneity, all cities will have the same $SMB_j(N^{cp})$,and through the absence of an integer problem $N^{cp}/N_{TOT} = J^*$, the optimal number of cities. With homogeneity, and equal allocation of N will satisfy (FP), however the global optimum also maximizes each individual SAB.

From the city planner system of cities welfare can be improved by moving a resident away from the inferior city k to the superior city j, since $SAB_j(N_j^{cp}) > SAB_k(N_k^{cp})$, therefore $N_j^{fp} > N_j^{cp}$.[6] Figure 1 illustrates this difference: here SAB_1 is given by the solid curve and SAB_2, by the outside option, where point A gives the city planner solution, and point C, the federal planner. Figure 2 illustrates an example with 2 cities where city 1 is superior to city 2, and where $N_1^{fp} > N_1^{cp} = N_{TOT}/2 = N_2^{cp} > N_2^{fp}$. The city planner solution is given by points A and B, and the federal planner by point C. In both figures, the deadweight loss of the city planner solution is equal to the area between the SMP curves, from the efficient to the inefficient population levels.

2.2. **Private Ownership and Individual Incentives.** Residency in a city may affect the income or the amenities of residents in other cities because of across-city spill-overs. This produces a wedge between the average social and private benefit of residing in a city, which we define as the across-city wedge:

$$ACW_j(N) \equiv SAB_j(N) - PAB_j(N) \qquad \text{(ACW)}$$

where $PAB_j(N)$ is the private average benefit, which like the SAB is assumed to be twice continuously differentiable, strictly quasi-concave, and single-peaked as in (3). The across-city wedge may distort PAB relative to SAB even if the magnitude of the wedge is zero. For example, federal income taxes create a wedge between the social and private average benefits by distorting the marginal benefit of income by $(1-\tau)$. Even if the amount each city was taxed was rebated lump sum back to the city the distortion would remain because the observed marginal effect is distorted. We normalize the sum of across-city wedges to zero such that the sum of the private average benefits equal the sum of social average benefits.[7]

$$\sum_j N_j ACW_j(N_j) = 0 \qquad (5)$$

In the competitive equilibrium all individuals are mobile across cities. Therefore the across-city mobility condition, equation CE, and the stability condition, equation 6, characterize the competitive equilibrium. The across-city mobility condition ensures no individual can be made better off by

[6] When $N_k^{cp} = 0$, then $N_j^{fp} \geq N_j^{cp}$ trivially.

[7] This normalization assumes that the across-city wedge is shifts production but does not create or destroy production in the economy.

moving across cities. The stability condition rules out population distributions that are not robust to a slight deviation.[8]

$$PAB_j(N_j^{ce}) = PAB_k(N_k^{ce}) \quad \text{for all inhabited cities } j \text{ and } k \tag{CE}$$

$$\frac{\partial PAB_j(N_j^{ce})}{\partial N} + \frac{\partial PAB_k(N_k^{ce})}{\partial N} \leq 0 \tag{6}$$

The competitive equilibrium may not maximize the welfare of city residents. We define a political equilibrium, denoted with "pe", as the population level existing residents or city developers would limit the size of a city to maximize private average benefit levels within a city. The political equilibrium is given by point D in figure 1 and is analogous to the city planner optimum, except that across-city externalities are internalized.[9]

$$PAB_j(N_j^{pe}) = PMB_j(N_j^{pe}) \tag{PE}$$

Conditions CE, 6, and PE characterize the competitive and political equilibria on the intensive margin. This paper adopts the extensive margin algorithm created in Seegert (2011a) where individuals create and populate cities in a two stage game.[10] In the first stage individuals decide simultaneously whether to create a city and which city to create. In the second stage individuals in the competitive equilibrium move across cities such that conditions CE and 6 hold and in the political equilibrium such that condition PE holds. The subgame perfect equilibrium of this dynamic game characterizes the extensive margin which can be found by backward induction.

2.3. **Private versus Efficient Incentives.** The competitive equilibrium condition CE equalizes the private average benefits while the federal planner equalizes the social marginal benefits across cities. Equation 7 decomposes the difference between the efficient and the competitive allocation into the difference between the social and private average benefits, defined as the across-city wedge, and the difference between the marginal and average benefit, defined as the within-city wedge.

[8]The stability condition can be replaced by restricting the set of allowable equilibria to be trembling hand perfect, as demonstrated by Seegert (2011a).
[9]As with the city-planner problem, the political equilibrium is subject to integer problems.
[10]For a dynamic model of city formation see Seegert (2011b).

Collectively these two wedges define the private-social wedge.

$$(7) \qquad Private - Social\, Wedge = SMB_j(N) - PAB_j(N) = WCW_j(N) + ACW_j(N)$$

These two wedges are illustrated in figure 3 to the right of N^{cp} where both wedges are positive. The previous literature emphasizes the locational efficiency gains from eliminating the within-city wedge however this point no longer holds in a system of heterogeneous cities, see result one. With homogeneity and ignoring integer problems, locational inefficiencies arise because all points to the right of $N^{cp} = N^{fp}$ are potentially stable competitive equilibria while no points to the left are. This leads to the textbook maxim that "cities are not too small" (O'Sullivan 2009) while they can be too big. However, result two demonstrates this point breaks down when across-city wedges exist.

RESULT 2: *If the across-city wedge is increasing with population and cities are homogeneous then the stable competitive equilibrium is inefficiently small.*

In the competitive equilibrium cities are created until adding a new city would lower the shared private average benefit. When cities are homogeneous this implies the unique competitive equilibrium is the political equilibrium depicted as point B in figure 4. The efficient population occurs at A because homogeneity implies $N^{cp} = N^{fp}$. If the across-city wedge is increasing in population point B is to the left of point A. Therefore the competitive equilibrium, at point B, is smaller than the efficient population level, given by point A.

Despite the fact that across-city wedges distort the private average benefit total production in the system of cities remains constant.[11] If a single city is given the population N^{cp} its PAB will be given by point C which is lower than its SAB, and its across-city wedge given by the distance between A and C. If all cities coordinate to achieve point A the across-city wedge will cause the PAB curve to rise. In this scenario each city would benefit from limiting its population and attain point D to maximize its private average benefit while still receiving the spillover benefits from larger cities. Yet, if all cities did this the equilibrium will return to point B as the spill-overs are lost and the PAB curve shifts back down.

[11]Federal income taxes are an intuitive example of an across-city wedge that distorts the economies of scale within a city but that could be rebated lump sum to the cities such that the total benefit produced within a city is retained.

COROLLARY 2: *If the across-city wedge is increasing with population and cities are homogeneous then the competitive equilibrium produces too many cities.*

When cities are homogeneous the federal planner's optimum is the city planner's optimum and the number of cities the federal planner produces is $\mathcal{J}^{fp} = N_{tot}/N^{cp}$. As noted above the competitive equilibrium produces cities with populations that equal the political equilibrium which produces $\mathcal{J}^{ce} = N_{tot}/N^{pe}$. Therefore $N^{pe} < N^{cp}$ implies that $\mathcal{J}^{ce} > \mathcal{J}^{fp}$.

RESULT 3: *If the private-social wedge at the federal planner optimum is larger for superior cities then the competitive equilibrium will produce superior cities that are inefficiently small.*

Let cities be ordered by their superiority such that city 1 is superior to city 2 and city i is superior to city j. Assume toward contradiction that the competitive equilibrium population is larger than the federal planner population for some city i while the reverse is true for some city j such that city i is superior to city j. This condition can be written as $PAB_i(N_i^{fp}) > PAB_i(N_i^{ce})$ and $PAB_j(N_j^{fp}) < PAB_j(N_j^{ce})$ because the stability condition ensures populations are to the right of the peak of the private average benefit.

$$
\begin{aligned}
PAB_j(N_j^{fp}) &> PAB_i(N_i^{fp}) && \text{by assumption } PSW_i > PSW_j. \\
&> PAB_i(N_i^{ce}) && \text{by assumption toward contradiction.} \\
&= PAB_j(N_j^{ce}) && \text{definition competitive equilibrium.} \\
&> PAB_j(N_j^{fp}) && \text{by assumption toward contradiction.}
\end{aligned}
$$

Contractiction

Therefore when the private-social wedge at the federal planner optimum is larger for superior cities, cities $\{1, 2, ...h\}$, for some $1 \leq h \geq J$, in the competitive equilibrium are inefficiently small while cities $\{h + 1, 2, ...J\}$ are inefficiently large.

The following section creates a parametric system where economies of scale, urban costs, and the private-social wedge are modeled explicitly to determine under what conditions superior cities are inefficiently small in the competitive equilibrium. Result three demonstrates that the private-social wedge being larger for superior cities is a sufficient condition for superior cities to be inefficiently small. The parametric model uses the canonical Alonso-Muth-Mills monocentric city model to provide insights into result three, explicitly showing how federal income taxes and land rent produce an across-city wedge that can lead to inefficiently small superior cities in the competitive equilibrium.

3. Parametric System of Monocentric Cities

3.1. **City Structure, Commuting, Production, and Natural Advantages.** In each city, individuals reside around a central business district (CBD) where all urban production takes place. The city expands radially from the CBD with the conventional assumptions that urban costs are a function of distance z from the CBD. Each resident demands a lot size with a fixed area, normalized to one, so that a city of radius $\underline{z}$ contains a population $N = \pi(\underline{z})^2$.

The urban costs in the city are modeled as a time cost of commuting. The time an individual uses to commute comes out of the single unit of labor the individual supplies to the market. An individual who lives at a distance z supplies $h(z) = 1 - \tilde{c}_h z^{\chi_h}$ units of labor where $\tilde{c}_h$ is a positive scalar and χ_h is the nonnegative elasticity of the time cost of commuting with respect to distance. The aggregate labor supply in a circular city is given by $H(N) = N - c_h N^{1+\phi_h}$ where $c_h \equiv \tilde{c}_h \pi^{-1/2}(1+\phi_h)^{-1}$ and $\phi_j = 2\chi_h$ is the elasticity with respect to population. We extend traditional models that implicitly assume the elasticities with respect to distance, $\chi_h = 1$ by allowing it to be flexible. This flexibility accounts for fixed costs, variable density, and other factors that cause the observed elasticity to differ from unity. Additional urban costs such as a material cost of commuting and a depreciation of average land quality within the city are easily included according to equation 8 where w is the wage in the city and I represents the number of urban costs that are denoted in terms of the numeraire. These additional costs are left out of this section for notational ease but are included in the calibrated section.

$$c_h N^{\phi_h} w + \sum_i^I c_i N^{\phi_i} \tag{8}$$

The wage reflects the scale economies within the city and are modeled with an agglomeration parameter α following Dixit (1973) but which encompasses local information spill-overs and search and matching economies as reviewed in Duranton and Puga (2004).

Aggregate city production is $F(A_j, N) = A_j N^\alpha H(N)$, where A_j is the natural advantage of city j in productivity. The local scale economies are given by N^α, with $\alpha \in (0, 1/2)$ which are external to firms but internal to cities, such that firms exhibit constant returns but cities exhibit increasing returns. Therefore firms make zero profit and pay a wage $w = A_j N^\alpha$.

Individuals consume land, the produced good x which is tradeable across cities and has a price normalized to one, and the level of quality of life amenities within the city, Q_j. Utility is given by $U(x, Q_j)$, which is strictly increasing and quasi-concave in both arguments. The level of quality of life amenities is assumed to be uniform within a city and independent of city size.[12] It is convenient to write utility $U(x_j, Q_j) = U(x_j) + x_j^Q$ where x_j^Q is the compensating differential in terms of the numeraire.

3.2. **Planned Economies.** The city planner and federal planner tradeoff the economies of scale and urban costs within cities, though with different objectives. The city planner chooses the population for their city that maximizes the social average benefit at the point at which the economies of scale exactly equal the urban costs.[13]

$$SAB(A_j, N) \equiv A_j N_j^\alpha \left(1 - c_h N_j^{\phi_h}\right) + x_j^Q \tag{9}$$

The population at the peak of the social average benefit occurs at the point where the social marginal benefit intersects the social average benefit. The social marginal benefit is the sum of four terms; FMP is the marginal product that accrues to the firm; AE_j is the agglomeration externality, which goes to firms for which the household does not work; and CCE_j is the increase in average urban

[12]The model is robust to allowing quality of life amenities to depend on population.

[13]The microfounded social average benefit satisfies the three assumptions in the theory section that it is twice continuously differentiable, strictly quasi-concave, and single peaked.

costs.

$$SMB_j = \underbrace{A_j N_j^{\alpha}}_{\text{FMP}} + \underbrace{\alpha A_j N_j^{\alpha}}_{\text{AE}} - \underbrace{c_h A_j N_j^{\alpha} N_j^{\phi_h}}_{\text{CCE}} + x^Q \tag{10}$$

The federal planner concerned with maximizing the total benefit across cities equalizes the social marginal benefit across all cities. The difference between the federal planner and the city planner is the difference between the marginal and the average benefit defined as the within-city wedge.

$$WCW_j = \alpha A_j N_j^{\alpha} - \alpha A_j c_h N_j^{\alpha+\phi_h} - \phi_h A_j c_h N_j^{\alpha+\phi_h} \tag{11}$$

3.3. **Private Ownership and Individual Incentives.** With private ownership, individuals receive income from labor and land, and pay for taxes, rent, and tradable consumption. Firms pay a wage $w_j = A_j N_j^{\alpha}$ per labor unit, because factor and output markets are competitive, and a worker at distance z supplies $h(z)$ units of labor. Labor income is taxed at the federal rate of $\tau \in [0, 1]$ leaving workers with $(1 - \tau)A_j N^{\gamma} h(z)$.[14] Federal taxes are redistributed in the form of federal transfers T_j, which may be location dependent. When federal transfers are not tied to local wage levels, federal taxes turn a fraction τ of labor income into a common resource, reducing individuals' incentive to move to areas with high wages.[15]

The rent gradient within the city is determined by the within-city mobility condition which states in equilibrium the location costs, the urban costs plus the land rent, must be equal across all

[14]It is appropriate to use the marginal tax rate since we are considering marginal changes in labor income due to migration decisions. See Albouy (2009) for further discussion.

[15]Empirically, Albouy (2009) finds that federal transfers are not strongly correlated with wage levels in the United States, however Albouy (2012) finds that they are negatively related in Canada, increasing the size of the across-city fiscal spill-overs.

distances z within a city.[16][17]

$$r_j(z) = w\tilde{c}_h(\underline{z}^{\chi_h} - z^{\chi_h}) \tag{12}$$

The rent at the central business district, $r_j(0)$, gives the full location cost. The rent gradient declines to $r_j(\underline{z}_j) = 0$ at the edge of the city, where we normalize the opportunity cost of land to zero.[18] The rental income of residents in city j is

$$R_j = (1-\rho)\bar{r}_j + \rho\bar{R} \tag{13}$$

where $\bar{R} = \frac{1}{N^{tot}}\sum_{j=0}^{J} N_j\bar{r_j}$ is the average rent paid in all cities, and $\rho \in [0,1]$ is an exogenously fixed parameter that captures the proportion of an individual's portfolio that is diversified across all cities, as opposed to the land holdings only within the city the individual lives. Much of the previous literature has focused on the special case where $\rho = 0$ implying individuals receive the average rental income in the city they live in. This assumption while seemingly innocuous actually imposes unrealistic distortions in mobility across cities. For example, a new migrant to city j inherits a free plot of land at the average distance and gives up any other land holdings without payment. Consequently, this assumption provides a perverse incentive for individuals to move to cities with high average rent because they inherit the land for free. When $\rho = 1$, migrants to a city have to pay rent on any plot they occupy, but still receive income from land, albeit in an amount unrelated to their location decision. This assumption treats individuals anonymously and causes migrants to pay rent to access the advantages of a city. As ρ increases a higher share of rent is

[16]Because land is not used in production, wages do not negatively capitalize consumption amenities as in Roback (1982) – see Albouy (2009) for details. However, when not all sites are inhabited, individuals may choose to reside in areas with high Q_j but low A_j which can produce a negative correlation between wages and consumption amenities.

[17]

$$\begin{aligned}\text{Location Cost} &= w\tilde{c}_h z^{\chi_h} + \tilde{c}_m z^{\chi_m} + \tilde{c}_l z^{\chi_l} + r(z)\\ &= w\tilde{c}_h \underline{z}^{\chi_h} + \tilde{c}_m \underline{z}^{\chi_m} + \tilde{c}_l \underline{z}^{\chi_l}\\ &= r(0) \quad \text{Downtown rent}\end{aligned}$$

[18]More generally, we discuss land rents that are differential land rents. Assuming that the opportunity cost of land is greater than zero adds little to the model unless the opportunity cost varies with Q or A. For instance it may be possible that sunnier land is more amenable to urban residents, but also contributes to agricultural productivity, raising the opportunity cost as well. Given the low value of agricultural land relative to residential land, these effects are likely to be of small consequence.

redistributed across cities, rather than only within the city, and land income can be thought of as a common (federal) resource.[19]

Income net of location costs is equal for all individuals within a city causing them to consume the same level of the tradeable good x and the quality of life consumption x^Q. This level of consumption is defined as the private average benefit within the city.

$$PAB_j = (1-\tau)A_j N_j^{\alpha}\left(1-(1+\rho\phi_h)c_h N^{\phi_h}\right) + \rho\bar{R} + T_j + x_j^Q \tag{14}$$

The competitive equilibrium is characterized by the across-city mobility condition which ensures individuals do not have an incentive to move. Therefore in the competitive equilibrium the private average benefit is equal across all cities. The political equilibrium is defined as the peak of the private average benefit which occurs at the point that the private marginal benefit intersects the private average benefit. When cities are heterogeneous in their production and quality of life amenities the political equilibrium and competitive equilibrium differ.[20]

3.4. **Private versus Efficient Incentives.** Notice that when ρ and τ equal zero the private average benefit, equation 14 equals the social average benefit, equation 1. In this case the population allocation of the city planner and political equilibrium are the same but may not be efficient as they may differ from the federal planner's population allocation. When ρ or τ are not zero some of the benefit produced within a city is distributed across all cities either through tax transfers or land rent income. In this case the social average benefit and private average benefit will differ by

[19]If migrants owned plots of land in an origin city, they would still sell the land when moving to the destination city, since they can only live in one city at a time. This would unnecessarily complicate the analysis through income effects, and require us to consider the origin as well as destination of migrants. The situation with $\rho = 1$ may also be characterized as one of a migrant from a typical city in the economy, as $\bar{R}$ denotes the average rent on a plot of land anywhere. One could also assume that land is owned by the federal government or absentee landlords. In these cases rental earnings are the same and zero for all individuals.

[20]The difference between the private average benefit and the private marginal benefit is defined as the private within-city wedge.

$$\text{Private Marginal Product} =$$

$$(\alpha+1)A_j N_j^{\gamma}(1-\tau) - (1+\alpha+\phi_h)(1-\tau)\frac{1+\rho\phi_h}{1+\phi_h}c_h A_j N_j^{\phi_h+\alpha} + T_j + \rho\bar{R}$$

$$\text{PWCW} = \alpha(1-\tau)A_j N_j^{\alpha} - (\alpha+\phi_h)(1-\tau)\frac{1+\rho\phi_h}{1+\phi_h}c_h A_j N_j^{\phi_h+\alpha}$$

the across-city wedge.

$$\text{Across-City Wedge} = \tau A_j N_j^{\alpha} - \tau(1+\rho\phi_h)c_h A_j N_j^{\alpha+\phi_h} + (\rho\phi_h)c_h A_j N_j^{\alpha+\phi_h}$$

The federal planner equalizes the social marginal benefit across cities while the competitive equilibrium equalizes the private average benefit across cities. The difference between the social marginal benefit and the private average benefit is defined as the social-private wedge. The private-social wedge is the combination of the within-city wedge and the across-city wedge.

$$\begin{aligned}\text{Private-Social Wedge} &= SMB_j - PAB_j = WCW_j + ACW_j \\ &= (\alpha+\tau)A_j N_j^{\alpha} - (\tau(1+\rho\phi_h) + \alpha + \phi_h(1-\rho))c_h A_j N_j^{\alpha+\phi_h} - \rho\bar{R} - T_j\end{aligned}$$

From this equation we can solve for the efficient governmental transfer; $T_j = AE_j + \tau A_j N_j^{\alpha} - \bar{r}_j + \rho\left(\bar{r}_j - \bar{R}\right)$. This transfer subsidizes the agglomeration externality AE_j and punishes for higher urban costs represented by the average rent $\bar{r}_j$. In addition the transfer rebates the fiscal externality the city provides to the common resource through taxes, $\tau A_j N_j^{\alpha}$ and land rent, $\rho\left(\bar{r}_j - \bar{R}\right)$.[21] When ρ and τ equal zero the across-city wedge is zero and the private-social wedge equals the within-city wedge.

$$\begin{aligned}\text{Private-Social Wedge}(\rho = o, \tau = 0) &= SMB_j - PAB_j = WCW_j \\ &= \underbrace{\alpha A_j N_j^{\alpha} - \alpha c_h A_j N_j^{\alpha+\phi_h}}_{\text{AE}} - \underbrace{\phi_h c_h A_j N_j^{\alpha+\phi_h}}_{\text{Average Rent}}\end{aligned}$$

In this case the private-social wedge equals the agglomeration externality, AE, minus the average rent in the city. This result is the Henry George theorem (Arnott and Stiglitz, 1979) which states that land taxes are a sufficient tax to produce the optimal level of public good. In this model the public good is the agglomeration externality given by AE. In this case without a land tax population could grow to any population level greater than the city planner's optimum as individuals consider the average and not marginal benefit within the city. The confiscatory land tax limits the

[21]In a closed-city context, Wildasin (1985) notes that the time costs of commuting are implicitly deducted from federal taxes, although the material costs are not, and argues that taxes lead to excessive sprawl by reducing the time-cost of commuting. This mechanism does not work in a closed-city setting with fixed lot sizes, but it does matter in an open-city setting by leveling the slope of the wage gradient, causing it to hit zero at a further distance, implying a larger population.

competitive equilibrium population size to the city planner level. The literature has focused on this condition because when cities are homogenous (and there is no across-city wedge) confiscatory land taxes provide the efficient allocation of population. However, if cities are heterogeneous with respect to production and consumption amenities and all cities impose confiscator land taxes the competitive equilibrium population levels are inefficiently small for the superior cities, see result one.

When taxes or intercity land income are introduced into the model the private-social wedge is the combination of the across-city and within-city wedge and therefore is no longer the simple combination of the public good and land rents. In a system of heterogeneous cities the superior cities will be undersized in the competitive equilibrium if the private-social wedge is increasing with the level of amenities provided, by result three. Taking the partial derivative of the private-social wedge with respect to the production amenity level A_j provides a partial equilibrium condition for when this condition and therefore result three holds.[22][23]

$$\frac{\alpha+\tau}{1-\rho(1-\tau)} \geq \frac{\phi_h c_h N^{\phi_h}}{(1-c_h N^{\phi_h})}$$

$$\tau\big|_{\rho=0} > \frac{\phi_h c_h N_j^{\phi_h}}{1-c_h N_j^{\phi_h}} - \alpha$$

$$\rho\big|_{\tau=0} > 1 - \frac{\alpha(1-c_h N_j^{\phi_h})}{\phi_h c_h N_j^{\phi_h}}$$

In the parametric example the sufficient condition from result three holds when the tax rate, τ, or the land income portfolio diversification parameter, ρ, exceed their threshold values given in

[22]In the calibration section a condition is provided from taking the total derivative.

[23]

$$\begin{aligned}
PSW &= (\alpha+\tau)A_j N_j^{\alpha} - (\tau(1+\rho\phi_h)+\alpha+\phi_h(1-\rho))c_h A_j N_j^{\alpha+\phi_h} - \rho\bar{R} - T_j \\
0 &> \frac{\partial PSW}{\partial A_j} \\
&= (\alpha+\tau)N_j^{\alpha} - (\tau(1+\rho\phi_h)+\alpha+\phi_h(1-\rho))c_h N_j^{\alpha+\phi_h} \\
0 &< (\alpha+\tau) - (\tau(1+\rho\phi_h)+\alpha+\phi_h(1-\rho))c_h N_j^{\phi_h} \\
\phi_h c_h N^{\phi_h}(1-\rho(1-\tau)) &< (\alpha+\tau)(1-c_h N^{\phi_h}) \\
\frac{\phi_h c_h N^{\phi_h}}{(1-c_h N^{\phi_h})} &< \frac{\alpha+\tau}{1-\rho(1-\tau)}
\end{aligned}$$

equation 3.4. The following section calibrates this parametric model to determine in a realistic environment whether taxes and land rent income are large enough forces to cause superior cities to become undersized.

4. Calibrated Model

4.1. **Calibration.** To test whether the private-social wedge satisfies the conditions in result 3 the model is calibrated using data from the Census Bureau, the Bureau of Labor Statistics, the American Community Survey (ACS), the Survey of Income and Program Participation (SIPP), and empirical studies by Rosenthal and Strange (2004); Albouy and Ehrlich (2011). The model is fully calibrated by nine parameters. The economies of scale in the model are calibrated by the agglomeration factor α, the population of the typical city, and the wage in the typical city. The urban costs in the model are split between commuting costs as a fraction of income and the elasticities with respect to population ϕ_i, where we consider three urban costs the time commuting cost, the material commuting cost, and the land depreciation cost.

According the to bureau of labor statistics May 2009 Occupational Employment and Wage Estimates in the United States the average annual salary is $\$43,460$. From Rosenthal and Strange (2004) survey on agglomeration they define a consensus range between .03 and .08, from which α is chosen to equal .05. The typical urban resident, the median resident, lives in Cleveland, OH with a population of $2,091,286$ according to the census bureau's annual estimates of population. From these three points the scalar A is found by taking the average annual wage and dividing by the typical city size to the agglomeration parameter α, $A = \frac{\text{Average Annual Wage}}{\text{Typical City Size}^{\alpha}}$.

About 10 percent of the working day and 5 percent of income is spent commuting according to the American Community Survey and Survey of Income and Program Participation. The authors' calculations find the elasticity of commuting with respect to population to be .1 implying $\phi_h = \phi_m =$ .1. The cost parameters are found by setting $c_h N^{\phi_h} = .1$ and $c_m N^{\phi_m} = .045 *$Average Annual Wage. The land depreciation elasticity and cost parameters are calculated to match the land rent gradient and land share of income which by the authors' calculation are .216 and .05 respectively.

As a robustness check the elasticity and and cost parameters for the land depreciation urban cost is calculated for different land rent gradients and land share of incomes. In table XX the land share of income is increased from 2.5% to 6% holding fixed the land rent gradient at .216. As the land share of income is increased the elasticity ϕ_l decreases and the cost parameter c_l increases. In addition the within-city wedge decreases, the across-city wedge increases, and the resulting private-social wedge decreases. In table XX the land share of income is increased holding fixed the land gradient at .5 and all of the previous results hold.

In table XX the elasticity of land value with respect to population is varied from .2 to .7 holding the land share of income fixed at 4.4%. As the elasticity increases ϕ_l increases and c_l decreases. The within-city wedge in levels is flat but in percentage decreases, the across-city wedge increases, and the resulting private-social wedge increases. In table XX the elasticity of land value with respect to population is increased over the same range with the land share of income fixed at 2.5% and all of the previous results hold.

4.2. **Calibrated Microfounded Model.** The superior cities in a system of heterogeneous cities will be undersized in the competitive equilibrium if the private-social wedge is increasing with the level of amenities within the city. In the micro-foundation section a partial equilibrium condition was derived by taking the partial derivative of the private-social wedge. In this section the calibration produces a range of values for τ and ρ such that private-social wedge is increasing with the level of amenities.

$$d\text{Private-Social Wedge} : \frac{\partial PSW}{\partial A} dA + \frac{\partial PSW}{\partial N} dN > 0$$

Given the calibration $\frac{\partial PSW}{\partial A} > 0$ for all values of $\rho \in [0, 1]$ and $\tau \in [0, 1]$. Allowing $dA > 0$ and assuming that $dN > 0$ then $\frac{\partial PSW}{\partial N} > 0$ is a sufficient condition for the private-social wedge to be larger for superior cities.

4.3. **Calibrated System of Heterogeneous Cities.** In this section we simulate a system of heterogenous cities using the calibrated model. The simulation demonstrates how the private-social wedge skews the distribution of population across cities (intensive margin) and the distribution of cities that are inhabited (extensive margin). When the private-social wedge is large the competitive equilibrium will inhabit more cities and underpopulate them relative to the federal planner. The misallocation of population in the competitive equilibrium leads to a deadweight loss of $170 billion or around 4% of income with the baseline calibration.

A. *City Formation.* The distribution of population across cities is calculated following Seegert (2011a) which focuses on the impact of migration constraints on the distribution of cities. The process of creating and inhabiting cities is done with a two-stage dynamic game where the resulting population distribution is a subgame perfect equilibrium. In the first stage the federal planner decides how many cities to inhabit. In the second stage the federal planner decides the population distribution across the cities equalizing the social marginal benefit. By backward induction the federal planner chooses the number of cities in the first stage that maximizes total production given the population distribution that will obtain in the second stage.

In the competitive equilibrium's first stage individuals simultaneously decide whether to create a new city in which they must live or wait and migrate to an existing city in stage two. In the second stage individuals simultaneously decide which city to live in. By backward induction the resulting distribution of population in the second stage will equalize the private average benefit, otherwise some individual could have done better and moved to the city with the larger private average benefit. In the first stage individuals considering the resulting distribution of population in the second stage continue to create new cities to maximize the resulting equalized private average benefit.

B. *Heterogeneity Calibration.* The heterogeneity in city amenities is calculated using the actual distribution of cities in the United States. The distribution of amenities is calculated to provide each city in the data the same level of private average benefit. This distribution is then used to determine the amenity levels for the next 200 hypothetical cities'.

The actual distribution of cities in the United States follows Zipf's law. The underlying economics of why the distribution follows Zipf's law remains an open question. Krugman in his 1996 paper conjectures that the reason cities follow Zipf's law is that the underlying distribution of amenities follow Zipf's law. The simulated distribution of amenities in this paper support this conjecture as the distribution of amenities follows Zipf's law.

$$log(Rank) = 11.332 - \underset{(-77.06)}{1.073}\ log(population/1000) \tag{15}$$

$$log(Rank) = 280 - \underset{(-112.96)}{30.258}\ log(A_j) \tag{16}$$

C. *Extensive Margin Results.* The baseline calibration, $\rho = 1$ and $\tau = .33$, leads to a stark difference between the distribution of cities in the competitive equilibrium and the efficient allocation and is reported in table XX column 1. The competitive equilibrium inhabits 361 cities and the largest city is about 19 million. In contrast, the efficient distribution inhabits only 20 cities with the largest being 68 million. The different calibrations are reported in columns 2 through 7 and demonstrate this stark contrast is a result of the large wedge caused by $\rho = 1$ and $\tau = .33$ and a relatively low level of urban costs. The relatively low level of urban costs creates an incentive for the federal planner to create fewer cities with larger populations than the competitive equilibrium.

The creation and growth of cities is an important research area that is relatively understudied. The notable exceptions are Fujita, Anderson, and Isard (1978) which produces normative models, Seegert (2011a) discussed above, and Seegert (2011b) which creates a positive dynamic model based solely on individual incentives.

5. Conclusion

The above analysis does not prove that cities are necessarily too small, but it does call into question the necessity of cities being too large in an economy where federal taxes are paid and residential land must be purchased. As a result, the ability of local governments to reduce city sizes by restricting development through impact fees, green belts, and zoning may do much to reduce overall welfare, as they will likely neglect across-city spillovers, fiscal and otherwise, and allow a small minority to monopolize the best sites, forcing others to occupy naturally less superior sites.

Many other factors certainly play a role in determining efficient city sizes, among them, the ability of governments to provide adequate regulation, public goods, and infrastructure to make a large city function well. This may be a particular challenge in developing countries, where rapidly growing cities suffer disproportionately from negative externalities such as dirty air, infectious disease, and debilitating traffic. Moreover, in these cities the marginal resident, perhaps a poor rural migrant, may not pay federal taxes or for their land costs by working in the informal sector and squatting on land they have no property rights to. Thus, the problem of under-sized cities may be a relatively new one historically, seen primarily in the developed world, but one that will become increasingly important as property rights develop, federal governments tax increasingly, and urbanization rises.

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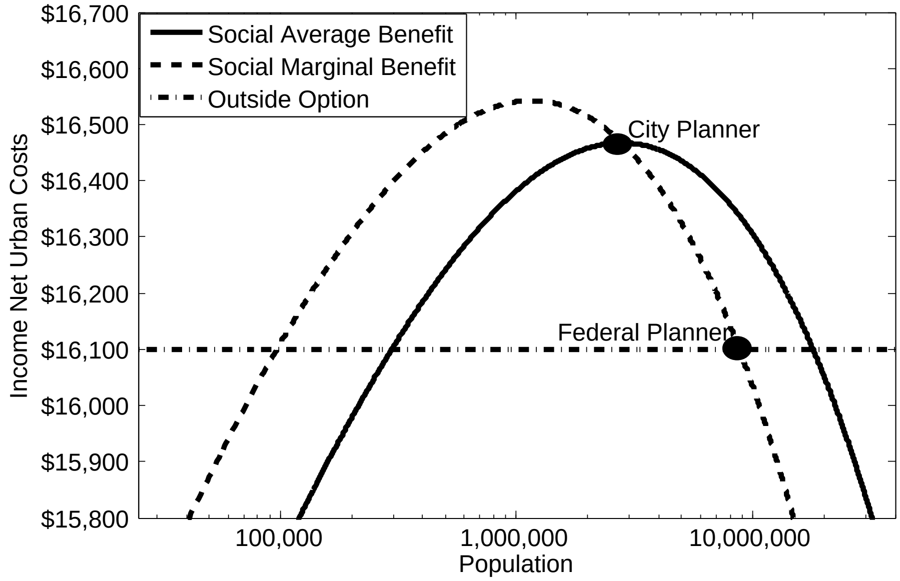
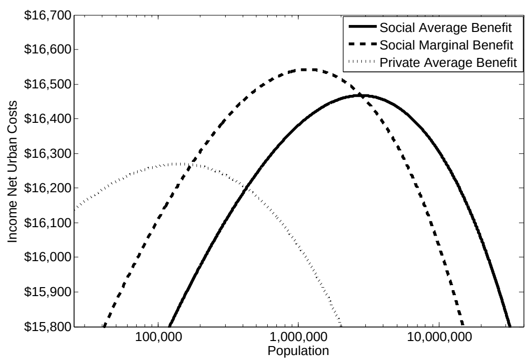
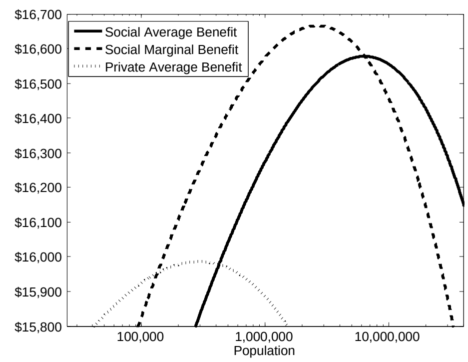
Figure 1. Social Average Benefit and Social Marginal Benefit

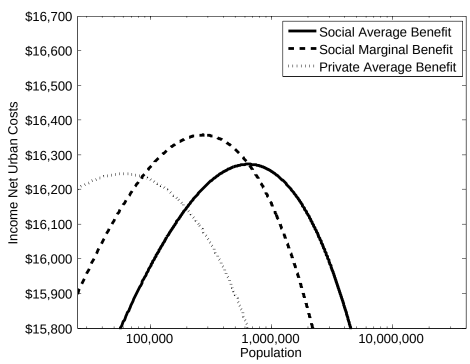




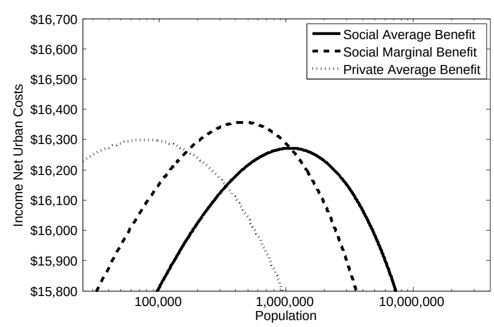
(a) Baseline Estimates



(b) Land Depreciation



(c) Commuting Costs Increased



(d) Agglomeration Increased

Figure 2. Robustness

FIGURE 3. Two City Example

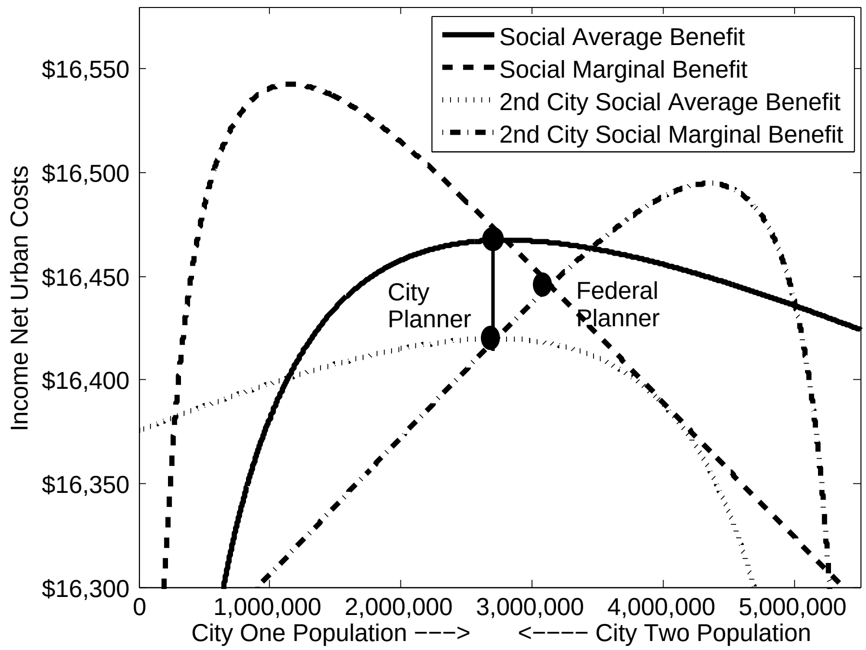
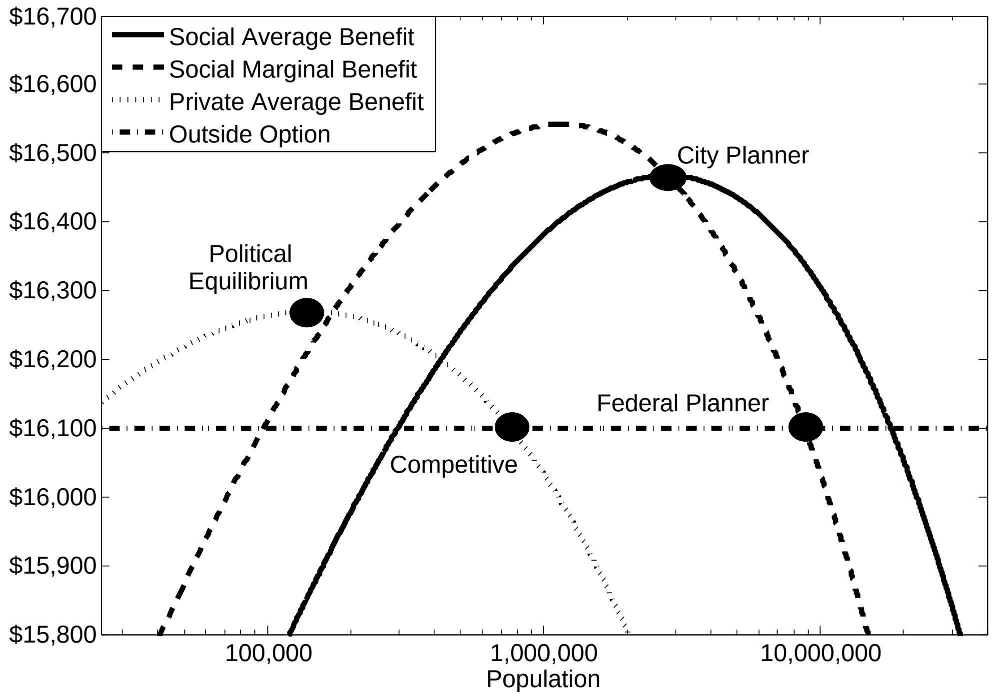
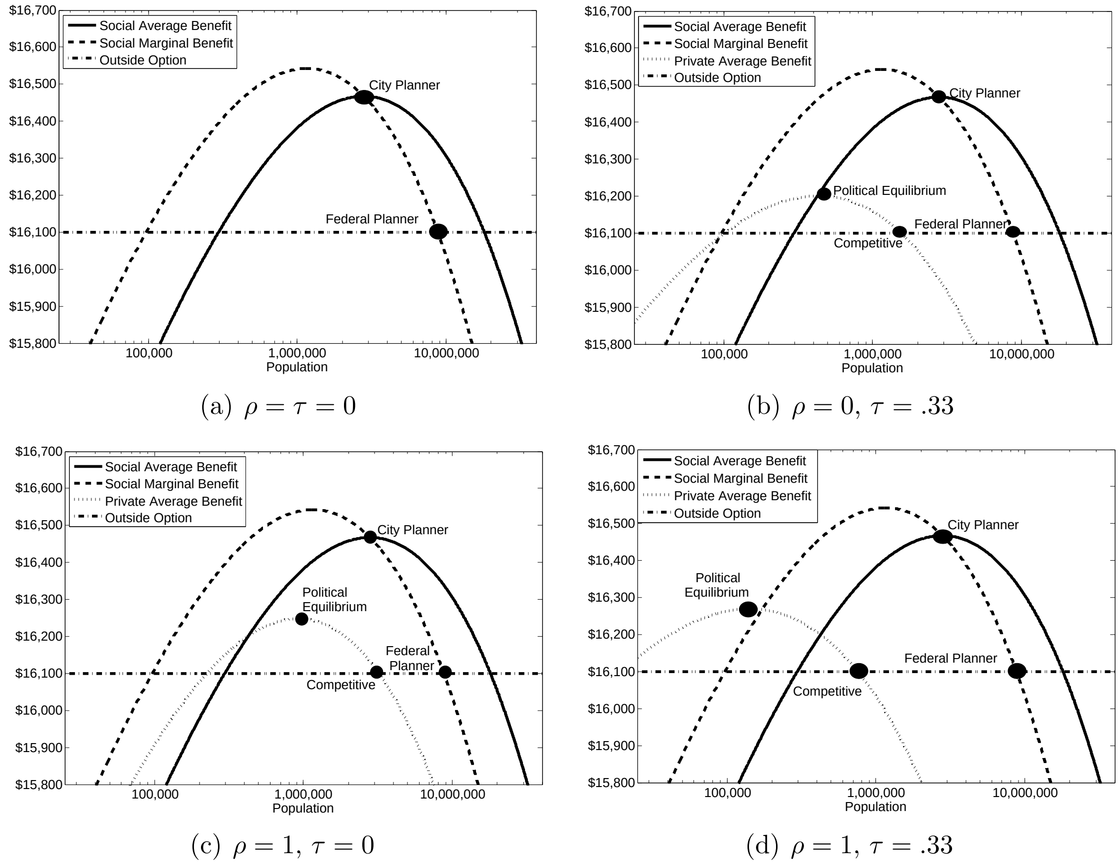


FIGURE 4. Equilibrium Concepts





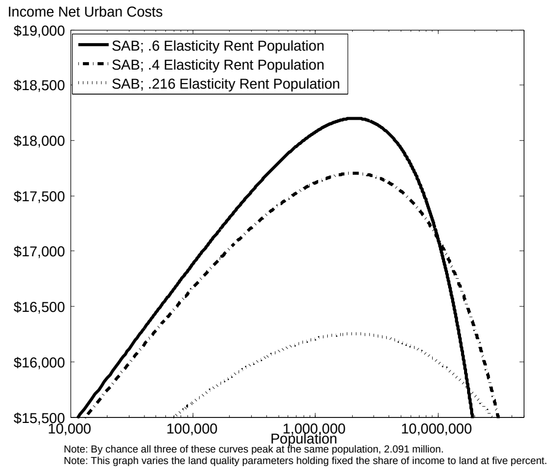
(a) $\rho = \tau = 0$

(b) $\rho = 0$, $\tau = .33$

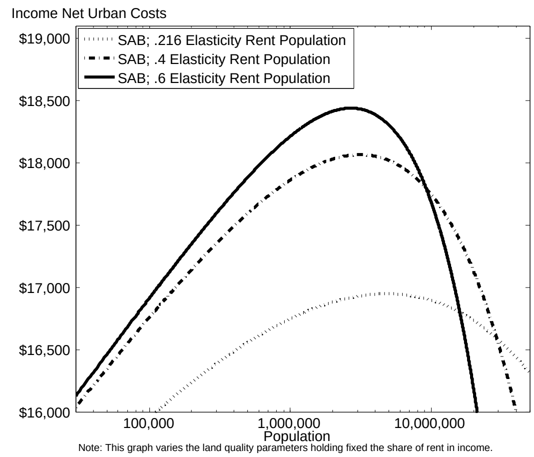
(c) $\rho = 1$, $\tau = 0$

(d) $\rho = 1$, $\tau = .33$

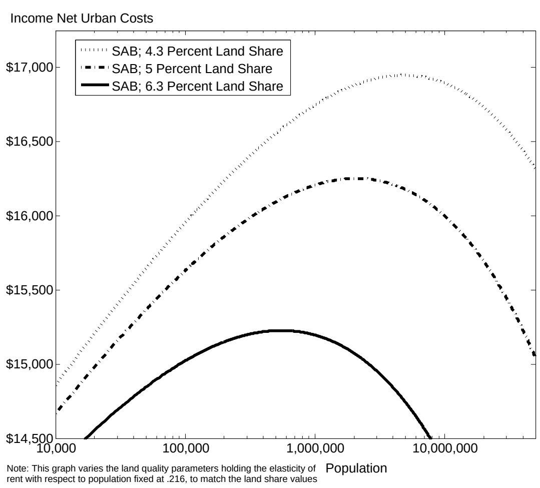
Figure 5. Across-City Wedge: Taxes and Land Rents



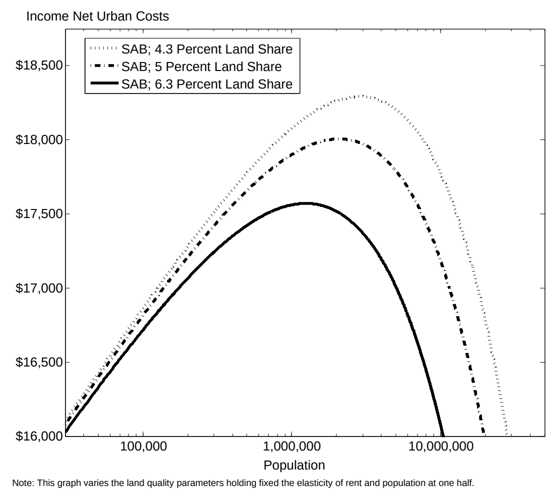
(a) Fixed: Share Income of Land



(b) Fixed: Share of Rent Income

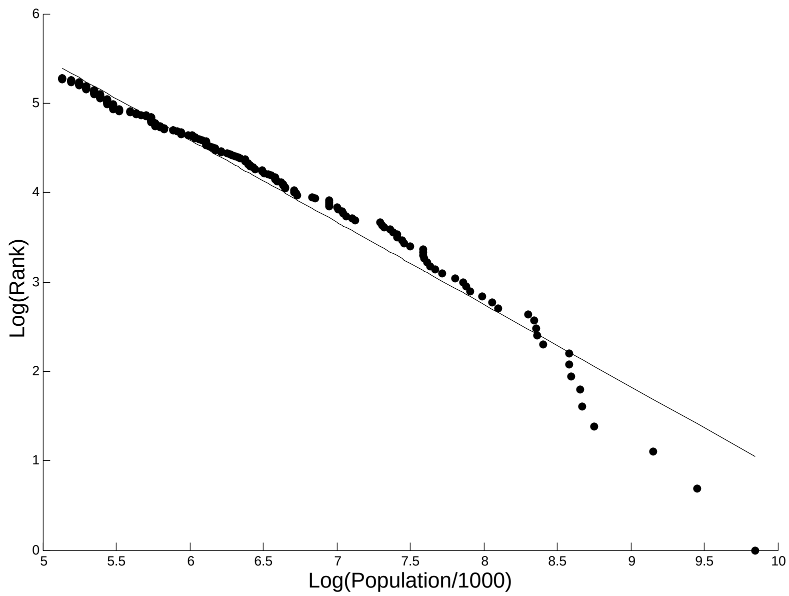


(c) Fixed: Elasticity of Rent at 0.216

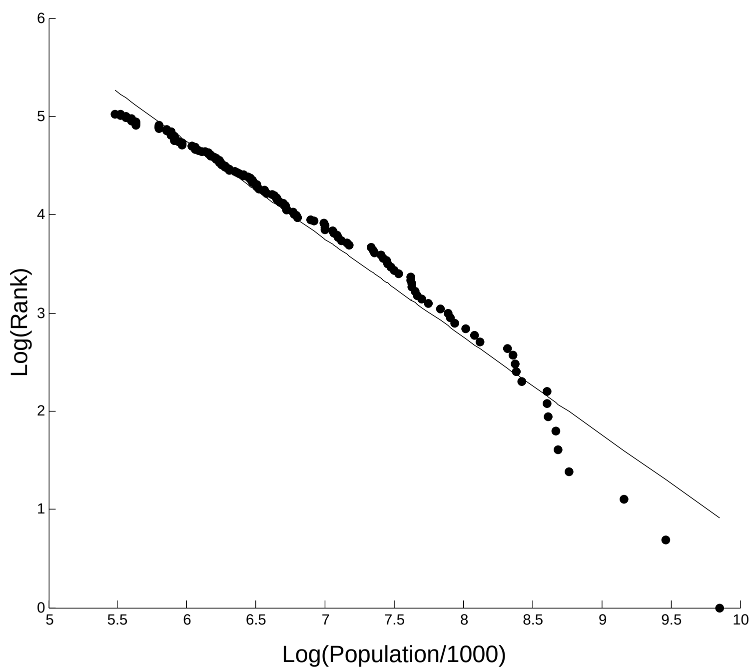


(d) Fixed: Elasticity Rent at 0.5

Figure 6. Robustness Vary Land Quality



(a) Zipf's Law With Production Amenities



(b) Zipf's Law With Quality of Life Amenities

Figure 7. Zipf's Law

Figure 8. Transit Time and Metro Population

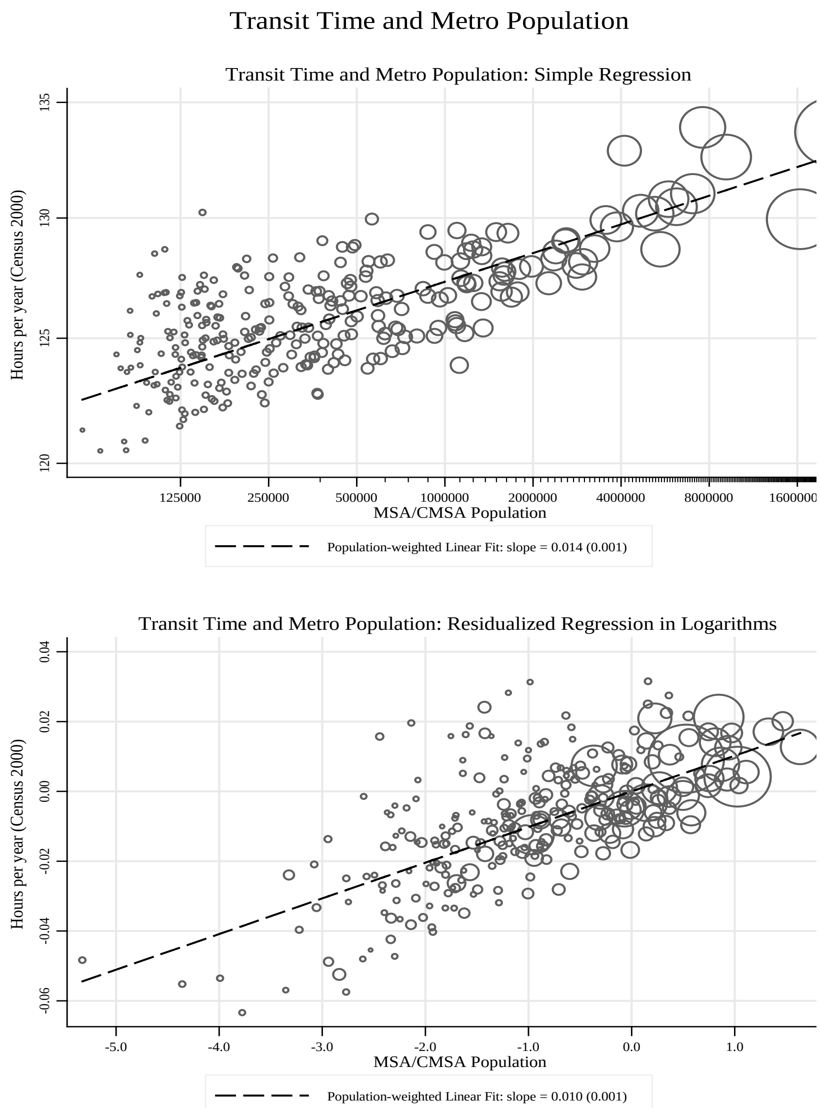


FIGURE 9. Average Annual Work Hours and Metro Population

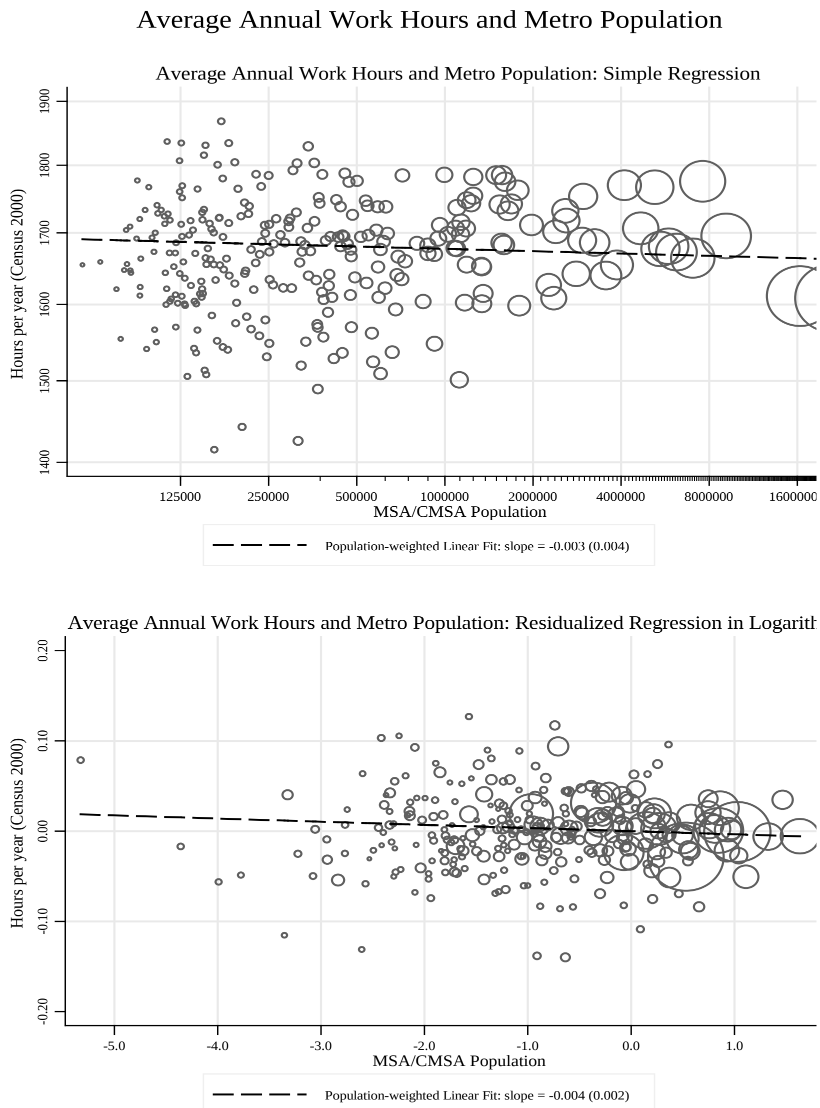


Figure 10. Inferred Land Rents and Metro Population

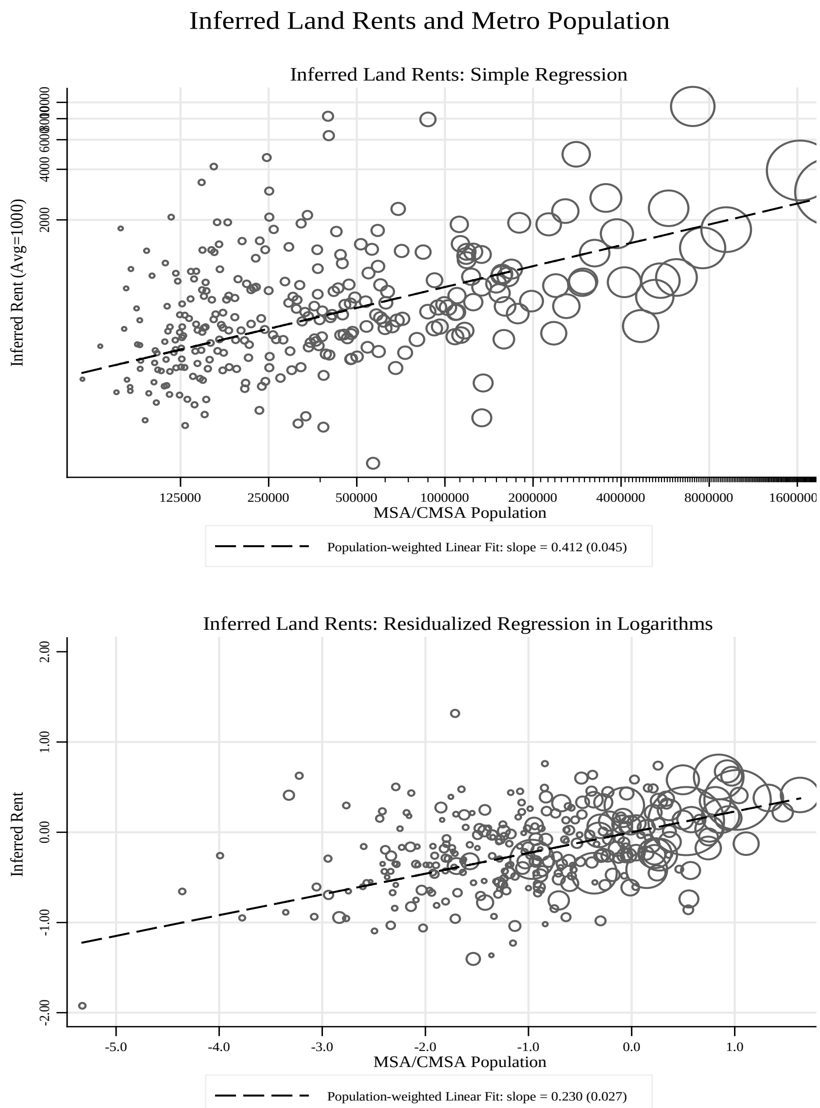


Figure 11. Measured Land Rents and Metro Population

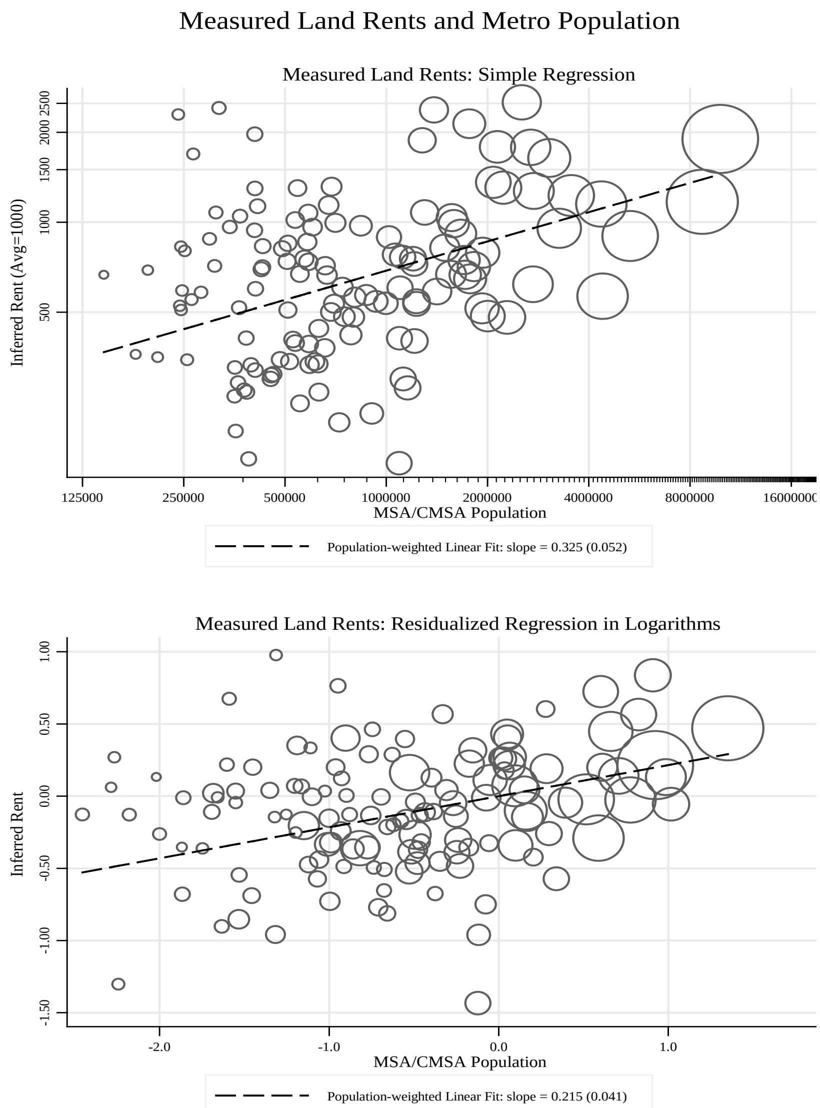


Table 1. Production Amenities

	Observed Case	Baseline Case	135 City Case	Large Urban Costs	$\phi = .5$ Case	$\tau = .2$ Case
Economic Parameters	(0)	(1)	(2)	(3)	(4)	(5)
Agglomeration Parameter γ		0.050	0.050	0.050	0.050	0.050
Commuting Parameter ϕ		0.100	0.100	0.100	0.100	0.100
Land Heterogeneity Parameter α		0.250	0.250	0.600	0.250	0.250
Avg. share of time lost to commuting		0.100	0.100	0.100	0.100	0.100
Avg. share of material cost of commuting		0.045	0.045	0.045	0.045	0.045
Heterogeneous Land weight		0.150	0.150	0.150	0.150	0.150
Size of typical city (millions)		2.091	2.091	2.091	2.091	2.091
Avg. value of labor ($1000s)		22.000	22.000	22.000	22.000	22.000
Fixed PAB		14500	14500	14500	12000	12000
Tax/Ownership Parameters						
Marginal tax rate τ		0.330	0.330	0.330	0.330	0.200
Land ownership parameter ρ		1.000	1.000	1.000	0.500	1.000
Implied Values						
Competitive Population Largest City	19,069,796	19,110,000	18,580,000	19,090,000	19,110,000	19,110,000
Efficient Population Largest City		68,620,000	65,640,000	38,000,000	51,270,000	44,550,000
Competitive Population Smallest City	55,176	80,000	120,000	100,000	80,000	90,000
Efficient Population Smallest City		1,560,000	4,580,000	560,000	670,000	550,000
Competitive Population Median City	244,694	240,000	400,000	260,000	240,000	250,000
Efficient Population Median City		7,365,000	9,100,000	2,600,000	3,195,000	3,040,000
Number of Competitive Cities	366	361	212	352	362	358
Number of Efficient Cities		20	14	51	38	42
Net Production Competitive Cities		3.747E+12	3.505E+12	3.749E+12	3.231E+12	3.097E+12
Net Production Efficient Cities		3.921E+12	3.646E+12	3.960E+12	3.196E+12	3.163E+12
Deadweight Loss Level (Difference)		1.732E+11	1.408E+11	2.107E+11	9.661E+10	6.600E+10
Deadweight Loss Percentage (Difference)		4.623%	4.018%	5.622%	2.990%	2.131%

Table 2. Quality of Life Amenities

	Observed Case	Baseline Case	Large Urban Costs	$\phi = .5$ Case	$\tau = .2$ Case	Large Costs $\phi = .5$
Economic Parameters	(0)	(1)	(2)	(3)	(4)	(5)
Agglomeration Parameter γ		0.050	0.050	0.050	0.050	0.050
Commuting Parameter ϕ		0.100	0.100	0.100	0.100	0.100
Land Heterogeneity Parameter α		0.250	0.600	0.250	0.250	0.600
Avg. share of time lost to commuting		0.100	0.100	0.100	0.100	0.100
Avg. share of material cost of commuting		0.045	0.045	0.045	0.045	0.045
Heterogeneous Land weight		0.150	0.150	0.150	0.150	0.150
Size of typical city (millions)		2.091	2.091	2.091	2.091	2.091
Avg. value of labor ($ 1000s)		22.000	22.000	22.000	22.000	22.000
Fixed PAB		15000	15000	15000	15000	15000
Tax/Ownership Parameters						
Marginal tax rate τ		0.330	0.330	0.330	0.200	0.330
Land ownership parameter ρ		1.000	1.000	0.500	1.000	0.500
Implied Values						
Competitive Population Largest City	19,069,796	19,250,000	19,160,000	19,220,000	19,220,000	19,020,000
Efficient Population Largest City		41,680,000	22,500,000	32,950,000	34,850,000	16,540,000
Competitive Population Smallest City	55,176	190,000	340,000	330,000	350,000	240,000
Efficient Population Smallest City		3,250,000	1,050,000	2,960,000	2,990,000	1,240,000
Competitive Population Median City	244,694	325,000	450,000	450,000	520,000	650,000
Efficient Population Median City		8,850,000	1,980,000	5,890,000	6,060,000	2,365,000
Number of Competitive Cities	366	300	234	234	219	152
Number of Efficient Cities		24	79	32	31	54
Net Production Competitive Cities		4.157E+12	4.136E+12	4.152E+12	4.070E+12	3.517E+12
Net Production Efficient Cities		4.244E+12	4.167E+12	4.210E+12	4.129E+12	3.526E+12
Deadweight Loss Level (Difference)		8.689E+10	3.082E+10	5.725E+10	5.862E+10	9.381E+09
Deadweight Loss Percentage (Difference)		2.090%	0.745%	1.379%	1.440%	0.267 %

Table 3. Land Share Robustness

	Benchmark case	Land Share 2.5 %	Land Share 4.3%	Land Share 5 %	Land Share 6.3%	Land Share 8%
	(1)	(2)	(3)	(4)	(5)	(6)
Economic Parameters						
Agglomeration Parameter γ	0.050	0.050	0.050	0.050	0.050	0.050
Commuting Parameterϕ	0.100	0.100	0.100	0.100	0.100	0.100
Land Heterogeneity Parameter α	0.250	0.286	0.246	0.241	0.235	0.217
Avg. share of time lost to commuting	0.100	0.100	0.100	0.100	0.100	0.100
Avg. share of material cost of commuting	0.045	0.045	0.045	0.045	0.045	0.045
Heterogeneous Land weight	0.150	0.056	0.146	0.182	0.248	4.018
Size of typical city (millions)	2.091	2.091	2.091	2.091	2.091	2.091
Avg. value of labor ($ 1000s)	22.000	22.000	22.000	22.000	22.000	22.000
Tax/Ownership Parameters						
Marginal tax rate τ	0.330	0.330	0.330	0.330	0.330	0.330
Land ownership parameter ρ	1.000	1.000	1.000	1.000	1.000	1.000
Implied Values						
Share of income to differential land rents s_R	0.044	0.025	0.043	0.050	0.063	0.800
Elasticity of land value to population $\varepsilon_{r,N}$	0.220	0.216	0.216	0.216	0.216	0.216
Commuting share of rent	0.328	0.580	0.337	0.290	0.230	0.018
Typical Within-City Social Wedge (SMB-SAB)	50.000	434.000	73.998	-65.999	-326.002	-15065.961
Typical Across-City Wedge (SAB-PAB)	1,349.62	1,222.20	1,338.29	1,383.24	1,466.65	6,188.39
Typical Social-Private Wedge (SMB-PAB)	1,399.62	1,656.19	1,412.29	1,317.24	1,140.65	-8,877.57
Typical Within-City Private Wedge (PMB-PAB)	1,247.45	1,717.55	1,279.79	1,109.55	793.39	-17,130.40
Top Within-City Social Wedge (SMB-SAB)	180.55	909.40	243.84	-16.67	-501.11	-28,017.56
Top Across-City Wedge (SMB-PAB)	2,922.56	2,450.29	2,871.94	3,037.40	3,345.25	20,843.49
Top Social-Private Wedge (SMB-PAB)	3,103.11	3,359.69	3,115.78	3,020.74	2,844.14	-7,174.07
Top Within-City Private Wedge (PMB-PAB)	1,233.08	2,121.51	1,316.95	1,000.80	412.41	-33,045.73
Typical City-Planner Population (millions)	2.802	49.703	3.268	1.440	0.399	0.001
Typical Political Equilibrium Population (millions)	0.142	3.554	0.161	0.066	0.016	0.001
Typical Within-City Social Wedge Percent (SMB-SAB)	0.304%	2.393%	0.448%	-0.416%	-2.220%	28.147%
Typical Across-City Wedge Percent (SAB-PAB)	8.199%	6.740%	8.102%	8.712%	9.988%	-11.561%
Typical Social-Private Wedge Percent (SMB-PAB)	8.503%	9.133%	8.550%	8.296%	7.768%	16.586%
Typical Within-City Private Wedge Percent (PMB-PAB)	7.579%	9.471%	7.748%	6.988%	5.403%	32.004%
Top Within-City Social Wedge Percent (SMB-SAB)	1.150%	4.805%	1.536%	-0.114%	-4.030%	24.376%
Top Across-City Wedge Percent (SMB-PAB)	18.620%	12.946%	18.095%	20.704%	26.905%	-18.135%
Top Social-Private Wedge Percent (SMB-PAB)	19.770%	17.750%	19.631%	20.591%	22.874%	6.242%
Top Within-City Private Wedge Percent (PMB-PAB)	7.856%	11.209%	8.298%	6.822%	3.317%	28.751%

Table 4. Land Share Robustness With Elasticity = 0.5

	Benchmark case	Land Share 2.5 %	Land Share 4.3%	Land Share 5 %	Land Share 6.3%	Land Share 8%
	(1)	(2)	(3)	(4)	(5)	(6)
Economic Parameters						
Agglomeration Parameter γ	0.050	0.050	0.050	0.050	0.050	0.050
Commuting Parameterϕ	0.100	0.100	0.100	0.100	0.100	0.100
Land Heterogeneity Parameter α	0.250	0.800	0.630	0.607	0.580	0.505
Avg. share of time lost to commuting	0.100	0.100	0.100	0.100	0.100	0.100
Avg. share of material cost of commuting	0.045	0.045	0.045	0.045	0.045	0.045
Heterogeneous Land weight	0.150	0.028	0.075	0.093	0.128	2.136
Size of typical city (millions)	2.091	2.091	2.091	2.091	2.091	2.091
Avg. value of labor ($ 1000s)	22.000	22.000	22.000	22.000	22.000	22.000
Tax/Ownership Parameters						
Marginal tax rate τ	0.330	0.330	0.330	0.330	0.330	0.330
Land ownership parameter ρ	1.000	1.000	1.000	1.000	1.000	1.000
Implied Values						
Share of income to differential land rents s_R	0.044	0.025	0.043	0.050	0.063	0.800
Elasticity of land value to population $\varepsilon_{r,N}$	0.220	0.500	0.500	0.500	0.500	0.500
Commuting share of rent	0.328	0.580	0.337	0.290	0.230	0.018
Typical Within-City Social Wedge (SMB-SAB)	50.000	434.000	74.000	-66.000	-325.996	-15066.028
Typical Across-City Wedge (SAB-PAB)	1,349.62	1,321.76	1,541.66	1,625.49	1,780.42	10,494.19
Typical Social-Private Wedge (SMB-PAB)	1,399.62	1,755.76	1,615.66	1,559.49	1,454.43	-4,571.83
Typical Within-City Private Wedge (PMB-PAB)	1,247.45	1,575.55	1,035.55	825.55	435.56	-21,674.50
Top Within-City Social Wedge (SMB-SAB)	180.55	-1,249.86	-2,392.91	-2,943.31	-3,999.11	-66,416.40
Top Across-City Wedge (SMB-PAB)	2,922.56	4,709.11	5,712.05	6,206.29	7,157.03	63,548.06
Top Social-Private Wedge (SMB-PAB)	3,103.11	3,459.25	3,319.15	3,262.98	3,157.92	-2,868.34
Top Within-City Private Wedge (PMB-PAB)	1,233.08	-2,090.37	-3,479.61	-4,271.53	-5,819.52	-99,359.33
Typical City-Planner Population (millions)	2.802	6.780	2.491	1.806	1.086	0.004
Typical Political Equilibrium Population (millions)	0.142	1.686	0.502	0.351	0.202	0.001
Typical Within-City Social Wedge Percent (SMB-SAB)	0.304%	2.314%	0.409%	-0.370%	-1.883%	124.267%
Typical Across-City Wedge Percent (SAB-PAB)	8.199%	7.047%	8.522%	9.121%	10.283%	-86.558%
Typical Social-Private Wedge Percent (SMB-PAB)	8.503%	9.361%	8.931%	8.751%	8.400%	37.709%
Typical Within-City Private Wedge Percent (PMB-PAB)	7.579%	8.401%	5.724%	4.632%	2.516%	178.775%
Top Within-City Social Wedge Percent (SMB-SAB)	1.150%	-7.082%	-16.066%	-21.413%	-34.524%	58.586%
Top Across-City Wedge Percent (SMB-PAB)	18.620%	26.684%	38.351%	45.152%	61.786%	-56.056%
Top Social-Private Wedge Percent (SMB-PAB)	19.770%	19.602%	22.285%	23.739%	27.262%	2.530%
Top Within-City Private Wedge Percent (PMB-PAB)	7.856%	-11.845%	-23.362%	-31.076%	-50.239%	87.645%

Table 5. Wedges Free Land Under Alternate Calibrations

	Benchmark case	Land Share 2.5 %	Land Share 4.3%	Land Share 5 %	Land Share 6.3%	Land Share 8%
	(1)	(2)	(3)	(4)	(5)	(6)
Economic Parameters						
Agglomeration Parameter γ	0.050	0.050	0.050	0.050	0.050	0.050
Commuting Parameterϕ	0.100	0.100	0.100	0.100	0.100	0.100
Land Heterogeneity Parameter α	0.250	0.286	0.246	0.241	0.235	0.217
Avg. share of time lost to commuting	0.100	0.100	0.100	0.100	0.100	0.100
Avg. share of material cost of commuting	0.045	0.045	0.045	0.045	0.045	0.045
Heterogeneous Land weight	0.150	0.056	0.146	0.182	0.248	4.018
Size of typical city (millions)	2.091	2.091	2.091	2.091	2.091	2.091
Avg. value of labor ($ 1000s)	22.000	22.000	22.000	22.000	22.000	22.000
Tax/Ownership Parameters						
Marginal tax rate τ	0.330	0.330	0.330	0.330	0.330	0.330
Land ownership parameter ρ	1.000	0.000	0.000	0.000	0.000	0.000
Implied Values						
Share of income to differential land rents s_R	0.044	0.025	0.043	0.050	0.063	0.800
Elasticity of land value to population $\varepsilon_{r,N}$	0.220	0.216	0.216	0.216	0.216	0.216
Commuting share of rent	0.328	0.580	0.337	0.290	0.230	0.018
Typical Within-City Social Wedge (SMB-SAB)	50.000	434.00	74.00	-66.00	-326.00	-15065.96
Typical Across-City Wedge (SAB-PAB)	1,349.62	1065.51	1065.51	1065.51	1065.51	1065.51
Typical Social-Private Wedge (SMB-PAB)	1,399.62	1499.51	1139.51	999.51	739.51	-14000.45
Typical Within-City Private Wedge (PMB-PAB)	1,247.45	1675.04	1315.04	1175.05	915.04	-13824.92
Top Within-City Social Wedge (SMB-SAB)	180.55	909.40	243.84	-16.67	-501.11	-28017.56
Top Across-City Wedge (SMB-PAB)	2,922.56	1836.12	1836.12	1836.12	1836.12	1836.12
Top Social-Private Wedge (SMB-PAB)	3,103.11	2745.52	2079.96	1819.45	1335.01	-26181.45
Top Within-City Private Wedge (PMB-PAB)	1,233.08	2112.62	1447.06	1186.56	702.12	-26814.34
Typical City-Planner Population (millions)	2.802	49.703	3.268	1.440	0.399	0.001
Typical Political Equilibrium Population (millions)	0.142	10.450	0.495	0.204	0.052	0.001
Typical Within-City Social Wedge Percent (SMB-SAB)	0.304%	2.393%	0.448%	-0.416%	-2.220%	28.147%
Typical Across-City Wedge Percent (SAB-PAB)	8.199%	5.876%	6.451%	6.711%	7.256%	-1.991%
Typical Social-Private Wedge Percent (SMB-PAB)	8.503%	8.269%	6.899%	6.295%	5.036%	26.156%
Typical Within-City Private Wedge Percent (PMB-PAB)	7.579%	9.237%	7.961%	7.400%	6.231%	25.828%
Top Within-City Social Wedge Percent (SMB-SAB)	1.150%	4.805%	1.536%	-0.114%	-4.030%	24.376%
Top Across-City Wedge Percent (SMB-PAB)	18.620%	9.701%	11.569%	12.516%	14.767%	-1.597%
Top Social-Private Wedge Percent (SMB-PAB)	19.770%	14.505%	13.105%	12.402%	10.737%	22.779%
Top Within-City Private Wedge Percent (PMB-PAB)	7.856%	11.162%	9.117%	8.088%	5.647%	23.330%

Table 6. Wedges No Tax Under Alternate Calibrations

	Benchmark case	Land Share 2.5 %	Land Share 4.3%	Land Share 5 %	Land Share 6.3%	Land Share 8%
	(1)	(2)	(3)	(4)	(5)	(6)
Economic Parameters						
Agglomeration Parameter γ	0.050	0.050	0.050	0.050	0.050	0.050
Commuting Parameterϕ	0.100	0.100	0.100	0.100	0.100	0.100
Land Heterogeneity Parameter α	0.250	0.329	0.257	0.249	0.240	0.218
Avg. share of time lost to commuting	0.100	0.100	0.100	0.100	0.100	0.100
Avg. share of material cost of commuting	0.045	0.045	0.045	0.045	0.045	0.045
Heterogeneous Land weight	0.150	0.039	0.127	0.162	0.228	3.997
Size of typical city (millions)	2.091	2.091	2.091	2.091	2.091	2.091
Avg. value of labor ($ 1000s)	22.000	22.000	22.000	22.000	22.000	22.000
Tax/Ownership Parameters						
Marginal tax rate τ	0.330	0.000	0.000	0.000	0.000	0.000
Land ownership parameter ρ	1.000	1.000	1.000	1.000	1.000	1.000
Implied Values						
Share of income to differential land rents s_R	0.044	0.025	0.043	0.050	0.063	0.800
Elasticity of land value to population $\varepsilon_{r,N}$	0.220	0.216	0.216	0.216	0.216	0.216
Commuting share of rent	0.328	0.580	0.337	0.290	0.230	0.018
Typical Within-City Social Wedge (SMB-SAB)	50.000	500.00	140.00	0.00	-260.00	-15000.03
Typical Across-City Wedge (SAB-PAB)	1,349.62	755.05	872.03	917.08	1000.58	5722.58
Typical Social-Private Wedge (SMB-PAB)	1,399.62	1255.06	1012.03	917.08	740.58	-9277.45
Typical Within-City Private Wedge (PMB-PAB)	1,247.45	2862.97	2425.21	2254.97	1938.81	-15985.07
Top Within-City Social Wedge (SMB-SAB)	180.55	997.17	340.29	80.54	-403.13	-27918.13
Top Across-City Wedge (SMB-PAB)	2,922.56	1226.60	1640.45	1805.25	2112.42	19609.38
Top Social-Private Wedge (SMB-PAB)	3,103.11	2223.76	1980.74	1885.79	1709.29	-8308.75
Top Within-City Private Wedge (PMB-PAB)	1,233.08	3720.61	2933.20	2618.50	2031.59	-31423.62
Typical City-Planner Population (millions)	2.802	50.000	4.880	2.091	0.553	0.001
Typical Political Equilibrium Population (millions)	0.142	26.013	1.627	0.689	0.180	0.001
Typical Within-City Social Wedge Percent (SMB-SAB)	0.304%	2.708%	0.829%	0.000%	-1.726%	28.234%
Typical Across-City Wedge Percent (SAB-PAB)	8.199%	4.090%	5.164%	5.643%	6.642%	-10.771%
Typical Social-Private Wedge Percent (SMB-PAB)	8.503%	6.798%	5.993%	5.643%	4.916%	17.462%
Typical Within-City Private Wedge Percent (PMB-PAB)	7.579%	15.508%	14.362%	13.875%	12.870%	30.088%
Top Within-City Social Wedge Percent (SMB-SAB)	1.150%	5.119%	2.066%	0.527%	-3.089%	24.425%
Top Across-City Wedge Percent (SMB-PAB)	18.620%	6.296%	9.958%	11.815%	16.188%	-17.156%
Top Social-Private Wedge Percent (SMB-PAB)	19.770%	11.415%	12.023%	12.342%	13.098%	7.269%
Top Within-City Private Wedge Percent (PMB-PAB)	7.856%	19.098%	17.805%	17.138%	15.568%	27.492%

Table 7. Wedges No Tax Free Land Under Alternate Calibrations

	Benchmark case	Land Share 2.5 %	Land Share 4.3 %	Land Share 5 %	Land Share 6.3 %	Land Share 8 %
Economic Parameters						
Agglomeration Parameter γ	(1)	(2)	(3)	(4)	(5)	(6)
Commuting Parameterϕ						
Land Heterogeneity Parameter α	0.050	0.050	0.050	0.050	0.050	0.050
Avg. share of time lost to commuting	0.100	0.100	0.100	0.100	0.100	0.100
Avg. share of material cost of commuting	0.250	0.329	0.257	0.249	0.240	0.218
Heterogeneous Land weight	0.100	0.100	0.100	0.100	0.100	0.100
Size of typical city (millions)	0.045	0.045	0.045	0.045	0.045	0.045
Avg. value of labor ($ 1000s)	0.150	0.039	0.127	0.162	0.228	3.997
Tax/Ownership Parameters	2.091	2.091	2.091	2.091	2.091	2.091
Marginal tax rate τ	22.000	22.000	22.000	22.000	22.000	22.000
Land ownership parameter ρ						
Implied Values	0.330	0.000	0.000	0.000	0.000	0.000
Share of income to differential land rents s_R	1.000	0.000	0.000	0.000	0.000	0.000
Elasticity of land value to population $\varepsilon_{r,N}$						
Commuting share of rent	0.044	0.025	0.043	0.050	0.063	0.800
	0.220	0.216	0.216	0.216	0.216	0.216
Typical Within-City Social Wedge (SMB-SAB)	0.328	0.580	0.337	0.290	0.230	0.018
Typical Across-City Wedge (SAB-PAB)						
Typical Social-Private Wedge (SMB-PAB)	50.000	500.00	140.00	0.00	-260.00	-15,000.03
Typical Within-City Private Wedge (PMB-PAB)	1,349.62	600.00	600.00	600.00	600.00	600.00
	1,399.62	1,100.00	740.00	600.00	340.00	-14,400.03
Top Within-City Social Wedge (SMB-SAB)	1,247.45	2,746.34	2,386.34	2,246.34	1,986.34	-12,753.70
Top Across-City Wedge (SMB-PAB)						
Top Social-Private Wedge (SMB-PAB)	180.55	997.17	340.29	80.54	-403.13	-27,918.13
Top Within-City Private Wedge (PMB-PAB)	2,922.56	600.00	600.00	600.00	600.00	600.00
	3,103.11	1,597.17	940.29	680.54	196.87	-27,318.13
Typical City-Planner Population (millions)	1,233.08	3,610.71	2,953.84	2,694.09	2,210.42	-25,304.58
Typical Political Equilibrium Population (millions)						
	2.802	50.000	4.880	2.091	0.553	0.001
Typical Within-City Social Wedge Percent (SMB-SAB)	0.142	50.000	4.880	2.091	0.553	0.001
Typical Across-City Wedge Percent (SAB-PAB)						
Typical Social-Private Wedge Percent (SMB-PAB)	0.304%	2.708%	0.829%	0.000%	-1.726%	28.234%
Typical Within-City Private Wedge Percent (PMB-PAB)	8.199%	3.250%	3.553%	3.692%	3.983%	-1.129%
	8.503%	5.959%	4.382%	3.692%	2.257%	27.104%
Top Within-City Social Wedge Percent (SMB-SAB)	7.579%	14.877%	14.132%	13.822%	13.185%	24.005%
Top Across-City Wedge Percent (SMB-PAB)						
Top Social-Private Wedge Percent (SMB-PAB)	1.150%	5.119%	2.066%	0.527%	-3.089%	24.425%
Top Within-City Private Wedge Percent (PMB-PAB)	18.620%	3.080%	3.642%	3.927%	4.598%	-0.525%
	19.770%	8.198%	5.708%	4.454%	1.509%	23.900%
	7.856%	18.534%	17.930%	17.632%	16.938%	22.138%

Table 8. Elasticity of Rent

	Benchmark case	Elasticity Rent, Pop 0.2	Elasticity Rent, Pop 0.216	Elasticity Rent, Pop 0.3	Elasticity Rent, Pop 0.4	Elasticity Rent, Pop 0.5
	(1)	(2)	(3)	(4)	(5)	(6)
Economic Parameters						
Agglomeration Parameter γ	0.050	0.050	0.050	0.050	0.050	0.050
Commuting Parameterϕ	0.100	0.100	0.100	0.100	0.100	0.100
Land Heterogeneity Parameter α	0.250	0.225	0.246	0.360	0.495	0.630
Avg. share of time lost to commuting	0.100	0.100	0.100	0.100	0.100	0.100
Avg. share of material cost of commuting	0.045	0.045	0.045	0.045	0.045	0.045
Heterogeneous Land weight	0.150	0.158	0.146	0.109	0.087	0.075
Size of typical city (millions)	2.091	2.091	2.091	2.091	2.091	2.091
Avg. value of labor ($ 1000s)	22.000	22.000	22.000	22.000	22.000	22.000
Tax/Ownership Parameters						
Marginal tax rate τ	0.330	0.330	0.330	0.330	0.330	0.330
Land ownership parameter ρ	1.000	1.000	1.000	1.000	1.000	1.000
Implied Values						
Share of income to differential land rents s_R	0.044	0.043	0.043	0.043	0.043	0.043
Elasticity of land value to population $\varepsilon_{r,N}$	0.220	0.200	0.216	0.300	0.400	0.500
Commuting share of rent	0.328	0.337	0.337	0.337	0.337	0.337
Typical Within-City Social Wedge (SMB-SAB)	50.000	73.998	73.998	74.001	74.000	74.000
Typical Across-City Wedge (SAB-PAB)	1,349.62	1,322.14	1,338.29	1,413.53	1,485.30	1,541.66
Typical Social-Private Wedge (SMB-PAB)	1,399.62	1,396.14	1,412.29	1,487.53	1,559.30	1,615.66
Typical Within-City Private Wedge (PMB-PAB)	1,247.45	1,293.55	1,279.79	1,207.55	1,121.55	1,035.55
Top Within-City Social Wedge (SMB-SAB)	180.55	322.41	243.84	-259.85	-1,120.58	-2,392.91
Top Across-City Wedge (SMB-PAB)	2,922.56	2,777.22	2,871.94	3,450.88	4,383.37	5,712.05
Top Social-Private Wedge (SMB-PAB)	3,103.11	3,099.63	3,115.78	3,191.02	3,262.79	3,319.15
Top Within-City Private Wedge (PMB-PAB)	1,233.08	1,441.22	1,316.95	484.76	-1,045.51	-3,479.61
Typical City-Planner Population (millions)	2.802	3.411	3.268	2.840	2.613	2.491
Typical Political Equilibrium Population (millions)	0.142	0.135	0.161	0.285	0.405	0.502
Typical Within-City Social Wedge Percent (SMB-SAB)	0.304%	0.455%	0.448%	0.427%	0.415%	0.409%
Typical Across-City Wedge Percent (SAB-PAB)	8.199%	8.127%	8.102%	8.155%	8.337%	8.522%
Typical Social-Private Wedge Percent (SMB-PAB)	8.503%	8.581%	8.550%	8.582%	8.752%	8.931%
Typical Within-City Private Wedge Percent (PMB-PAB)	7.579%	7.951%	7.748%	6.967%	6.295%	5.724%
Top Within-City Social Wedge Percent (SMB-SAB)	1.150%	2.052%	1.536%	-1.611%	-7.110%	-16.066%
Top Across-City Wedge Percent (SMB-PAB)	18.620%	17.673%	18.095%	21.391%	27.813%	38.351%
Top Social-Private Wedge Percent (SMB-PAB)	19.770%	19.725%	19.631%	19.780%	20.703%	22.285%
Top Within-City Private Wedge Percent (PMB-PAB)	7.856%	9.171%	8.298%	3.005%	-6.634%	-23.362%

Table 9. Elasticity of Rent With No Tax

	Benchmark case (1)	Elasticity Rent, Pop 0.2 (2)	Elasticity Rent, Pop 0.216 (3)	Elasticity Rent, Pop 0.3 (4)	Elasticity Rent, Pop 0.4 (5)	Elasticity Rent, Pop 0.5 (6)
Economic Parameters						
Agglomeration Parameter γ	0.050	0.050	0.050	0.050	0.050	0.050
Commuting Parameterϕ	0.100	0.100	0.100	0.100	0.100	0.100
Land Heterogeneity Parameter α	0.250	0.233	0.257	0.384	0.535	0.686
Avg. share of time lost to commuting	0.100	0.100	0.100	0.100	0.100	0.100
Avg. share of material cost of commuting	0.045	0.045	0.045	0.045	0.045	0.045
Heterogeneous Land weight	0.150	0.137	0.127	0.093	0.074	0.064
Size of typical city (millions)	2.091	2.091	2.091	2.091	2.091	2.091
Avg. value of labor ($ 1000s)	22.000	22.000	22.000	22.000	22.000	22.000
Tax/Ownership Parameters						
Marginal tax rate τ	0.330	0.000	0.000	0.000	0.000	0.000
Land ownership parameter ρ	1.000	1.000	1.000	1.000	1.000	1.000
Implied Values						
Share of income to differential land rents s_R	0.044	0.043	0.043	0.043	0.043	0.043
Elasticity of land value to population $\varepsilon_{r,N}$	0.220	0.200	0.216	0.300	0.400	0.500
Commuting share of rent	0.328	0.337	0.337	0.337	0.337	0.337
Typical Within-City Social Wedge (SMB-SAB)	50.000	140.00	140.00	140.00	140.00	140.00
Typical Across-City Wedge (SAB-PAB)	1,349.62	856.17	872.03	944.96	1012.76	1064.54
Typical Social-Private Wedge (SMB-PAB)	1,399.62	996.17	1012.03	1084.96	1152.76	1204.54
Typical Within-City Private Wedge (PMB-PAB)	1,247.45	2438.97	2425.21	2352.97	2266.97	2180.97
Top Within-City Social Wedge (SMB-SAB)	180.55	421.15	340.29	-190.47	-1136.43	-2599.50
Top Across-City Wedge (SMB-PAB)	2,922.56	1543.73	1640.45	2244.14	3257.91	4772.75
Top Social-Private Wedge (SMB-PAB)	3,103.11	1964.88	1980.74	2053.67	2121.47	2173.25
Top Within-City Private Wedge (PMB-PAB)	1,233.08	3061.89	2933.20	2046.46	333.24	-2537.26
Typical City-Planner Population (millions)	2.802	5.317	4.880	3.700	3.152	2.881
Typical Political Equilibrium Population (millions)	0.142	1.721	1.627	1.379	1.279	1.243
Typical Within-City Social Wedge Percent (SMB-SAB)	0.304%	0.840%	0.829%	0.795%	0.776%	0.766%
Typical Across-City Wedge Percent (SAB-PAB)	8.199%	5.140%	5.164%	5.364%	5.616%	5.827%
Typical Social-Private Wedge Percent (SMB-PAB)	8.503%	5.980%	5.993%	6.159%	6.392%	6.593%
Typical Within-City Private Wedge Percent (PMB-PAB)	7.579%	14.642%	14.362%	13.357%	12.570%	11.938%
Top Within-City Social Wedge Percent (SMB-SAB)	1.150%	2.578%	2.066%	-1.145%	-7.045%	-17.216%
Top Across-City Wedge Percent (SMB-PAB)	18.620%	9.448%	9.958%	13.495%	20.196%	31.610%
Top Social-Private Wedge Percent (SMB-PAB)	19.770%	12.026%	12.023%	12.350%	13.151%	14.393%
Top Within-City Private Wedge Percent (PMB-PAB)	7.856%	18.740%	17.805%	12.306%	2.066%	-16.804%

Table 10. Elasticity of Rent With Free Land Assumption

	Benchmark case (1)	Elasticity Rent, Pop 0.2 (2)	Elasticity Rent, Pop 0.216 (3)	Elasticity Rent, Pop 0.3 (4)	Elasticity Rent, Pop 0.4 (5)	Elasticity Rent, Pop 0.5 (6)
Economic Parameters						
Agglomeration Parameter γ	0.050	0.050	0.050	0.050	0.050	0.050
Commuting Parameterϕ	0.100	0.100	0.100	0.100	0.100	0.100
Land Heterogeneity Parameter α	0.250	0.225	0.246	0.360	0.495	0.630
Avg. share of time lost to commuting	0.100	0.100	0.100	0.100	0.100	0.100
Avg. share of material cost of commuting	0.045	0.045	0.045	0.045	0.045	0.045
Heterogeneous Land weight	0.150	0.158	0.146	0.109	0.087	0.075
Size of typical city (millions)	2.091	2.091	2.091	2.091	2.091	2.091
Avg. value of labor ($ 1000s)	22.000	22.000	22.000	22.000	22.000	22.000
Tax/Ownership Parameters						
Marginal tax rate τ	0.330	0.330	0.330	0.330	0.330	0.330
Land ownership parameter ρ	1.000	0.000	0.000	0.000	0.000	0.000
Implied Values						
Share of income to differential land rents s_R	0.044	0.043	0.043	0.043	0.043	0.043
Elasticity of land value to population $\varepsilon_{r,N}$	0.220	0.200	0.216	0.300	0.400	0.500
Commuting share of rent	0.328	0.337	0.337	0.337	0.337	0.337
Typical Within-City Social Wedge (SMB-SAB)	50.000	74.00	74.00	74.00	74.00	74.00
Typical Across-City Wedge (SAB-PAB)	1,349.62	1065.51	1065.51	1065.51	1065.51	1065.51
Typical Social-Private Wedge (SMB-PAB)	1,399.62	1139.51	1139.51	1139.51	1139.51	1139.51
Typical Within-City Private Wedge (PMB-PAB)	1,247.45	1315.04	1315.04	1315.05	1315.04	1315.04
Top Within-City Social Wedge (SMB-SAB)	180.55	322.41	243.84	-259.85	-1120.58	-2392.91
Top Across-City Wedge (SMB-PAB)	2,922.56	1836.12	1836.12	1836.12	1836.12	1836.12
Top Social-Private Wedge (SMB-PAB)	3,103.11	2158.53	2079.96	1576.26	715.54	-556.79
Top Within-City Private Wedge (PMB-PAB)	1,233.08	1525.63	1447.06	943.37	82.65	-1189.68
Typical City-Planner Population (millions)	2.802	3.411	3.268	2.840	2.613	2.491
Typical Political Equilibrium Population (millions)	0.142	0.429	0.495	0.783	1.025	1.194
Typical Within-City Social Wedge Percent (SMB-SAB)	0.304%	0.455%	0.448%	0.427%	0.415%	0.409%
Typical Across-City Wedge Percent (SAB-PAB)	8.199%	6.549%	6.451%	6.147%	5.981%	5.890%
Typical Social-Private Wedge Percent (SMB-PAB)	8.503%	7.004%	6.899%	6.574%	6.396%	6.299%
Typical Within-City Private Wedge Percent (PMB-PAB)	7.579%	8.083%	7.961%	7.587%	7.381%	7.269%
Top Within-City Social Wedge Percent (SMB-SAB)	1.150%	2.052%	1.536%	-1.611%	-7.110%	-16.066%
Top Across-City Wedge Percent (SMB-PAB)	18.620%	11.684%	11.569%	11.381%	11.650%	12.328%
Top Social-Private Wedge Percent (SMB-PAB)	19.770%	13.736%	13.105%	9.771%	4.540%	-3.738%
Top Within-City Private Wedge Percent (PMB-PAB)	7.856%	9.708%	9.117%	5.848%	0.524%	-7.988%

Table 11. Elasticity of Rent With No Tax and Free Land

	Benchmark case (1)	Elasticity Rent, Pop 0.2 (2)	Elasticity Rent, Pop 0.216 (3)	Elasticity Rent, Pop 0.3 (4)	Elasticity Rent, Pop 0.4 (5)	Elasticity Rent, Pop 0.5 (6)
Economic Parameters						
Agglomeration Parameter γ	0.050	0.050	0.050	0.050	0.050	0.050
Commuting Parameterϕ	0.100	0.100	0.100	0.100	0.100	0.100
Land Heterogeneity Parameter α	0.250	0.233	0.257	0.384	0.535	0.686
Avg. share of time lost to commuting	0.100	0.100	0.100	0.100	0.100	0.100
Avg. share of material cost of commuting	0.045	0.045	0.045	0.045	0.045	0.045
Heterogeneous Land weight	0.150	0.137	0.127	0.093	0.074	0.064
Size of typical city (millions)	2.091	2.091	2.091	2.091	2.091	2.091
Avg. value of labor ($ 1000s)	22.000	22.000	22.000	22.000	22.000	22.000
Tax/Ownership Parameters						
Marginal tax rate τ	0.330	0.000	0.000	0.000	0.000	0.000
Land ownership parameter ρ	1.000	0.000	0.000	0.000	0.000	0.000
Implied Values						
Share of income to differential land rents s_R	0.044	0.043	0.043	0.043	0.043	0.043
Elasticity of land value to population $\varepsilon_{r,N}$	0.220	0.200	0.216	0.300	0.400	0.500
Commuting share of rent	0.328	0.337	0.337	0.337	0.337	0.337
Typical Within-City Social Wedge (SMB-SAB)	50.000	140.00	140.00	140.00	140.00	140.00
Typical Across-City Wedge (SAB-PAB)	1,349.62	600.00	600.00	600.00	600.00	600.00
Typical Social-Private Wedge (SMB-PAB)	1,399.62	740.00	740.00	740.00	740.00	740.00
Typical Within-City Private Wedge (PMB-PAB)	1,247.45	2,386.34	2,386.34	2,386.34	2,386.34	2,386.34
Top Within-City Social Wedge (SMB-SAB)	180.55	421.15	340.29	-190.47	-1,136.43	-2,599.50
Top Across-City Wedge (SMB-PAB)	2,922.56	600.00	600.00	600.00	600.00	600.00
Top Social-Private Wedge (SMB-PAB)	3,103.11	1,021.15	940.29	409.53	-536.43	-1,999.50
Top Within-City Private Wedge (PMB-PAB)	1,233.08	3,034.70	2,953.84	2,423.07	1,477.11	14.04
Typical City-Planner Population (millions)	2.802	5.317	4.880	3.700	3.152	2.881
Typical Political Equilibrium Population (millions)	0.142	5.317	4.880	3.700	3.152	2.881
Typical Within-City Social Wedge Percent (SMB-SAB)	0.304%	0.840%	0.829%	0.795%	0.776%	0.766%
Typical Across-City Wedge Percent (SAB-PAB)	8.199%	3.602%	3.553%	3.406%	3.327%	3.284%
Typical Social-Private Wedge Percent (SMB-PAB)	8.503%	4.443%	4.382%	4.201%	4.103%	4.051%
Typical Within-City Private Wedge Percent (PMB-PAB)	7.579%	14.326%	14.132%	13.546%	13.232%	13.062%
Top Within-City Social Wedge Percent (SMB-SAB)	1.150%	2.578%	2.066%	-1.145%	-7.045%	-17.216%
Top Across-City Wedge Percent (SMB-PAB)	18.620%	3.672%	3.642%	3.608%	3.719%	3.974%
Top Social-Private Wedge Percent (SMB-PAB)	19.770%	6.250%	5.708%	2.463%	-3.325%	-13.243%
Top Within-City Private Wedge Percent (PMB-PAB)	7.856%	18.574%	17.930%	14.571%	9.157%	0.093%